SOLUTIONS MANUAL

10th–12th Grade | Algebra 2

Exercise Solutions

Answer Keys

Principles of Algebra 2

APPLIED ALGEBRA FROM A BIBLICAL WORLDVIEW

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General Grading Notes

The parent/teacher should go over these notes with each student.

Attempts have been made to make this *Solutions Manual* as easy to use as possible. We've designed it with homeschool parents in mind, knowing that not every parent is comfortable with algebra and thus trying to annotate and show exactly how multi-step problems were solved. **The final answer is bolded** and highlighted in grey. Students should compare the final answers with their final answers, as well as how the answer was derived. We've also included notes throughout with additional explanation or grading reminders.

Below are a few overall thoughts to keep in mind.

Different Methods

Note also that there is often more than one legitimate way to solve a problem. Unless specifically told to, the student does not need to approach a problem the same way the *Solutions Manual* does. Provided they get the correct answer, that is okay.

Lowercase Versus Uppercase Letters

Listing the answer as a v when the *Solutions Manual* lists a V is a mistake worth pointing out to the student. Each case is a different symbol; there are problems in physics that use both v and V to stand for different values. The student should get in the habit of using the case given in the problem.

Units of Measure

It is important for students to include units of measure when the *Solutions Manual* lists one as part of the bolded answer, as keeping track of units of measure is an important skill. So if the answer is listed as 10 ft and the student only wrote 10, we would suggest only giving partial credit.

Differences in Rounding

If students follow the "General Instructions for Students" on page 7 in the *Teacher Guide,* hopefully their answers will be exactly those in the *Solutions Manual*. However, if a student rounds before the last step, their answer could be slightly off. This isn't an issue — just check to make sure they solved the problem correctly and remind them not to round until the very end to preserve accuracy. An exception to this is when working with irrational numbers like π where we *have* to use a rounded value; if the student uses a different rounded value, their answer may be slightly different. This isn't an issue. Note also that in Chapter 12, students will be instructed to round population growth problems down to the nearest whole number, as a part of a person, animal, etc., doesn't make sense. They'll also be instructed in problems to round dollars to the nearest cent.

Open-Ended Questions

There are a few open-ended questions to get students really thinking about a problem or meaning. The answers to these may vary in wording but should convey the same main idea(s) as that listed in the *Solutions Manual*.

Graphing

On problems where students draw graphs, except when specified otherwise, the graphs are only supposed to be rough sketches of what the calculator shows. Their purpose is to make sure the student did actually graph the function, but the student does not need to stress about getting the sketch exact. In fact, unless students are told to use it, graph paper doesn't need to be used — just the basic shape of the graph needs drawn. If the graph is totally different in shape, have the student try zooming in or out on the calculator (see Lesson 7.3), as a different zoom level can sometimes result in the shape looking very different. Note also that students may label graphs differently than the *Solutions Manual*. For example, when graphing $f(d) = 5d$, the vertical axis could be labeled f (the letter to the left of the parentheses can be thought of as the output), $f(d)$ (in function notation, the entire expression on the left can be thought of as the output), or y (there's a convention to always label the vertical axis the y-axis). All of those labels are correct.

Crossing Out Values

Note that in simplifying units of measure, we've tried to show units that cross out in the *Solutions Manual* (i.e., $2\ \cancel{\text{ft}^2}\left(\frac{3\text{ lb}}{6\ \cancel{\text{ft}^2}}\right) = 1\text{ lb}$); however, sometimes units need simplified by remembering what the exponent means/how to work with exponents (i.e., $2\text{ ft}^3\left(\frac{3\text{ lb}}{6\text{ ft}^2}\right) = 6\text{ ft}^{3-2} \bullet \text{lb} = 1\text{ ft} \bullet \text{lb}$). Except for with rational functions (introduced in Chapter 11), we've not shown cross outs with unknowns; however, students may cross out where appropriate when solving.

Assigning a Grade

The grade column in the Suggested Schedule (p. 9–16) in the *Teacher Guide* is available for you to keep track of a student's grade should you choose to do so. Feel free to use whatever method for grading you've chosen to adopt, or to leave those columns blank if you prefer not to assign grades.

- ✔ **Worksheets** – Because some problems on the worksheets are designed to challenge students (thus building their critical thinking skills and giving them a greater glimpse into how algebra applies), we would suggest adopting a grading method that grades worksheets based on mastery in the end rather than the number initially correct. This frees students up to not panic over harder problems — it is okay if students get some of the application problems wrong at first or need help in solving. Worksheets should be viewed as learning aids to help build problem-solving skills and exercise the mathematical tools being taught rather than as something to panic over getting wrong. If students get problems wrong, they should be given an opportunity to try again (with some hints, if appropriate). One way to help them is to give them the answer for a problem they got wrong, but not the solution, and have them see if they can show you how they could find the answer. So long as they take the time to learn and get the problem correct (or demonstrate how to find the correct answer) the second time, it can be counted as right. (*Note:* Use your own discretion on this. If students are continuing to make careless errors in one particular area, start counting that wrong unless they get it right the first time.) A simple method for grading is to divide the total problems gotten right by the total number of problems on the worksheets in that chapter. Then multiply that by 100 — the result is the numerical grade for that chapter's worksheets.
- ✔ **Challenge Problems** – A handful of problems throughout the book are marked as challenge problems. At the parent's/teacher's discretion, these can be skipped, counted as bonus problems (worth extra points if gotten correct), or treated like regular problems. They are marked in the *Solutions Manual* with a for your reference.

- **Quizzes and Tests** – Specific grading suggestions to calculate a numerical grade (i.e., a 95, 80, etc.) are given on the quizzes and tests themselves. Note that many of these are purposefully easier than the worksheets, as students will have a second chance on the worksheet problems but not on the quizzes and tests.
- **Overall Grade** – Add up all the grades for each chapter's worksheets and divide that by 16 (the total number of chapters). Add up all the quiz grades and divide that total by 15 (the total number of quizzes) to find the average of all the quizzes. Add up all the test grades and divide that total by 4 (the total number of tests) to find the average of all the tests. Add these 3 averages together with the score of the final exam, and then divide that total by 4. That is the overall numerical grade.

  Example: If a student gets an average of 85 on all the quizzes, an average of 90 on all the tests, an average of 95 on the chapter's worksheets, and 98 on the final exam, find $85 + 90 + 95 + 98 = 368$. Then divide 368 by 4, getting a numerical grade of 92.
- **Letter Grades** – If you want to use a letter grade, you would translate the numerical grade into a letter grade using whatever system you choose. For example, you might make 90 and up an A, 80–89 a B, 70–79 a C, 60–69 a D, and anything lower an F.

Calculators

While students are allowed to use a calculator throughout the course, they should not use the calculator to simplify algebraic expressions. While some calculators can perform calculations with variables, we believe it's important for students to learn to do them themselves. If they master the skill, they should be able to perform many more problems much more quickly than they could with a calculator (where they have to input all the variables) — and they'll have a better understanding of the concepts. To make sure students really have learned to simplify expressions by themselves, it is suggested that, except where specified, on quizzes and tests students use only a scientific calculator that cannot simplify algebraic expressions if possible. If they must use a graphing calculator that can handle algebraic expressions and do such simplification, make sure they do not use that functionality.

Additional Resources and Corrections

Please see the Book Extras page on ChristianPerspective.net for links to helpful online resources, along with additional notes and information related to this course.

Different Formats of Answers

One of the challenges of grading algebra is that there are different ways to express the same thing. We have tried hard in the wording of problems and the listing of the solutions to avoid as much of this as possible in order to make it easier for non-mathematical teachers to grade. The "General Instructions for Students" in the *Teacher Guide* tells students about specific formats to use; if they're followed, it will eliminate some of these issues. It's still possible that some differences may slip through, though, so here is a list of some different-looking-but-equivalent-meaning answers you might encounter. Answers with the same meaning but a different format should still be counted as correct.

- **Fractions versus Decimals** – i.e., $\frac{5}{4}$ instead of 1.25. (If you divide 5 by 4, you get 1.25.)
- **Different Order in Quantities Being Multiplied** – i.e., $8ba$ or $b8a$ instead of $8ab$. (Order doesn't matter in multiplication, so all of these mean the same thing, but the general convention is to list the known value first, then the unknowns in alphabetical order, as in $8ab$.)

- **Units of Measure in Different Forms** – i.e., kg m instead of kg • m (the • sign is optional), and kg $\bullet \frac{\text{m}}{\text{s}}$ instead of $\frac{\text{kg} \bullet \text{m}}{\text{s}}$ (the kg in the numerator of the fraction means the same thing as it does when being multiplied by the fraction — see Lesson 1.6).
- **Negative Exponents Used Instead of Positive Exponents in the Denominator, or Vice Versa** – i.e., $8a^{-2}$ instead of $\frac{8}{a^2}$, or vice versa. (See Lesson 2.2 for how $8a^{-2}$ and $\frac{8}{a^2}$ mean the same thing.)
- **A Power of 1** – i.e., x^1 instead of x. (On this, though, encourage the student next time to simplify to x.)
- **Swapped Sides of the Equal Sign** – i.e., $8 = x$ and $x = 8$.
- **Functions Labeled Differently** – i.e., using the convention to use $f(x)$ to stand for the dependent variable instead of the letters given in the problem. Students should have to defend why the notation they used works. (See Lesson 7.2 and examples in the *Solution Manual* there.)
- **Complex Answers Listed as 1 Fraction Instead of 2** – i.e., $\frac{3 \pm \sqrt{3}i}{2}$ instead of $\frac{3}{2} \pm \frac{\sqrt{3}i}{2}$. (You will see the $\frac{3 \pm \sqrt{3}i}{2}$ listed as a previous step in the solution so will be able to tell if it is accurate.)
- **Approximating Factors or Roots of a Function** – i.e., writing ≈ 4.472 instead of $2\sqrt{5}$. (Both mean the same thing, but $2\sqrt{5}$ is an exact rather than approximate answer. From Worksheet 9.6 forward, students were instructed to not approximate factors/roots unless told to find an approximate answer, so remind them of this, as it will make grading easier and give them more practice simplifying roots. But so long as they can show how they could rewrite in the format in the *Solutions Manual*, the problem can still be counted as correct. Note that students should leave their answer with the root simplified as much as possible without approximating — i.e., $\sqrt{20}$ should be listed as $2\sqrt{5}$.)
- **Transformed Functions Listed Differently** – It is not a problem if a student lists a transformed function (covered in Chapter 14) differently than the *Solutions Manual*, so long as they can show how their answer is the same as that in the *Solutions Manual*. However, remind students of the instructions given on Worksheet 14.2A (or of the instructions given for that particular problem), as following those should make their answer match that in the *Solutions Manual*.

Equation/Relationship/Function

In the notes next to solutions, you'll notice that sometimes we say "equation given," and other times "relationship given" or "function given." Most of the equations and relationships are also functions, so often a different word could have been used. The main point is simply that the equation shown was given to the student in the problem or text.

Notes on Quizzes and Tests

Unless otherwise indicated or you as the parent/teacher decide to allow it, students should not consult resources or notes while completing a quiz or test. Instructions regarding calculator use are given on each quiz/test.

Worksheet 1.1

Note: See General Grading Notes on page 3–6 for important grading notes. Also make sure students have decided how to take notes/study for the course (see the grey box on Worksheet 1.1).

1. **Answers will vary**, but should include the name and description of one of the properties covered on pages 3–4 of the *Student Textbook:* Commutative Property of Addition and Multiplication, Associative Property of Addition and Multiplication, Identity Property of Multiplication, Identity Property of Addition, or a property of division.

2.

a. **Can't tell, as it depends on what *a* and *d* equal. If they are equal, then the expressions are equal. If not, they are not.**

b. **=**

c. **=**

d. **Can't tell, as if *x* is 0, the expressions are equal; otherwise, they are not.**

e. **=**

f. **=**

g. **=**

h. **Can't tell, as *A* and *a* are different placeholders, so we don't know if they are equal.** *Note:* This is an important thing to watch throughout the course! Students should always use the same capitalization given when writing unknowns.

3.

a. **f**

b. **c**

4. **Here, *x* can't equal 3, as that would have us dividing by 0 and division by 0 is not possible, as you can't divide something by nothing.**

5.

a. **180 ft**	
$P = 2l + 2w$	(formula from Appendix B)
$P = 2(60\text{ ft}) + 2(30\text{ ft}) = 120\text{ ft} + 60\text{ ft} = 180\text{ ft}$	(inserted values and simplified)
b. $\approx$ **21.991 in**	
$C = \pi d$	(formula from Appendix B)
$C = \pi(7\text{ in}) \approx 21.991\text{ in}$	(inserted values and simplified)

Worksheet 1.2

Note: Students were instructed to read the General Instructions for Students on pages 7–8 of the *Teacher Guide* and store it somewhere for easy reference.

1.	**Answers will vary**, but should include the name and description of one of the conventions mentioned in Lesson 1.2 of the *Student Textbook*: symbols, plus and minus signs, different ways of showing multiplication.	
2.		
	a. $>$	(8 times 8 on the left equals 64, which is greater than 60.)
	b. $<$	(52 is less than 80.)
	c. $=$	(because of the commutative property of multiplication)
3.		
	a. **100 m • kg**	(5 times 20 equals 100.)
	b. $\boldsymbol{4a}$	
	c. **90 m • s**	(6 times 15 equals 90.)
4.	$\approx$ **15.7 in *or* 15.708 in**	
	$C = \pi d$	(formula from Appendix B)
	$C = \pi(5\text{ in}) \approx 15.7$ in or 15.708 in, depending on value used for π	(inserted values and simplified)
5.	**A unit of measure; it is not italicized.** (Note that this convention might not be universal.)	
6.	**No, because a different case could mean a different value.** We need to use the same symbol (and the same case) to represent an unknown.	

Worksheet 1.3

Note: See the note in General Grading Notes about how it is okay if students list quantities being multiplied together in a different order. For example, *aby* could be listed *bay* or *yab* and still be correct, as order and grouping doesn't matter in multiplication.

1.		
	a. $\boldsymbol{\frac{11}{3}}$	
	b. $\boldsymbol{\frac{\frac{1}{5}}{\frac{2}{6}}}$	(This can simplify to $\frac{3}{5}$, but students were told not to simplify for these problems.)
	c. $\boldsymbol{\frac{8}{F}}$	
	d. $\boldsymbol{\frac{x}{2y}}$	

2.	**7; if $x = 7$, that would have us dividing by 0 and division by 0 is not possible, as you can't divide something by nothing.**

3.

a. $\mathbf{\frac{9}{5}}$	
b. $\mathbf{\frac{2}{5a}}$	
c. $\mathbf{\frac{y}{x}}$	

4.

a. $\mathbf{\frac{5x}{3}}$	
b. $\mathbf{\frac{aby}{c}}$	(Students may have arranged the numerator in a different order, which is fine.)

5.

a. $\mathbf{\frac{2}{15}}$

$$\frac{2}{3}\left(\frac{1}{5}\right) = \frac{2(1)}{3(5)} = \frac{2}{15}$$

b. $\mathbf{\frac{8y}{3x}}$

$$\frac{8}{x}\left(\frac{y}{3}\right) = \frac{8y}{3x}$$

c. $\mathbf{\frac{3x}{2y}}$

$$\frac{\frac{x}{2}}{\frac{y}{3}} = \frac{x}{2}\left(\frac{3}{y}\right) = \frac{3x}{2y}$$

d. $\mathbf{\frac{3y}{2ax}}$

$$\frac{\frac{3}{x}}{\frac{2a}{y}} = \frac{3}{x}\left(\frac{y}{2a}\right) = \frac{3y}{2ax}$$

e. $\mathbf{\frac{2a}{3bx}}$

$$\frac{\frac{a}{b}}{\frac{3x}{2}} = \frac{a}{b}\left(\frac{2}{3x}\right) = \frac{2a}{3bx}$$

6. *Note:* Students were told to leave their answers as fractions on 6a and 6b.

a. $\mathbf{\frac{40\text{ mi}}{3\text{ gal}}}$

$\left(\frac{1}{3}\right)\frac{40\text{ mi}}{\text{gal}} = \frac{40\text{ mi}}{3\text{ gal}}$

b. $\mathbf{\frac{\$8}{3}}$

$\frac{2}{3}(\$4) = \frac{2(\$4)}{3} = \frac{\$8}{3}$

c. **150**

$\frac{50}{\frac{1}{3}} = 50\left(\frac{3}{1}\right) = 150$

d. $\boldsymbol{\frac{T}{s}}$

$n = \frac{T}{s}$ (The total yards divided by the portion of a yard we make each swatch will equal the number of swatches we can make.)

7. **8 kg • m**

$(2\text{ kg})(4\text{ m}) = 8\text{ kg} \bullet \text{m}$

Worksheet 1.4

1.

a. **4** (88 divided by 22 is 4.)

b. $\mathbf{\frac{8}{3}}$

$\frac{8a}{3a} = \frac{8\cancel{a}}{3\cancel{a}} = \frac{8}{3}$

c. $\boldsymbol{2x}$

$\frac{2xy}{y} = \frac{2x\cancel{y}}{\cancel{y}} = 2x$

2.

a. $\mathbf{\frac{4}{1}}$

b. $\boldsymbol{\frac{b}{1}}$

3.

a. $\boldsymbol{\frac{4x}{b}}$

b. $\dfrac{bc}{d}$	

4.

a. $\dfrac{2}{5}$	
$\dfrac{2}{3}\left(\dfrac{3}{5}\right) = \dfrac{2}{\cancel{3}}\left(\dfrac{\cancel{3}}{5}\right) = \dfrac{2}{5}$	
b. $\dfrac{1}{9}$	
$\dfrac{2}{6}\left(\dfrac{1}{3}\right) = \dfrac{2}{6}\left(\dfrac{1}{3}\right) = \dfrac{2}{18} = \dfrac{2}{2 \bullet 9} = \dfrac{\cancel{2}}{\cancel{2} \bullet 9} = \dfrac{1}{9}$	
c. $2x$	
$4\dfrac{x}{2} = \dfrac{4x}{2} = \dfrac{2 \bullet 2 \bullet x}{2} = \dfrac{\cancel{2} \bullet 2 \bullet x}{\cancel{2}} = 2x$	
d. $\dfrac{2x}{ab}$	
$\dfrac{\frac{2x}{3a}}{\frac{b}{3}} = \dfrac{2x}{3a}\left(\dfrac{3}{b}\right) = \dfrac{2x}{ab}$	

5.	**25 cm²**	*Note:* Students were told to give their answer in cm², as we haven't reviewed calculating units of measure.
	$A = \dfrac{1}{2}bh$	(formula from Appendix B)
	$A = \dfrac{1}{2}(10)(5) = 25$	(inserted values and simplified)

Worksheet 1.5

1. *Note:* Students should include the unit of measure.

a. $\dfrac{\$4}{3 \text{ dozen}}$ ***or*** $\dfrac{\$4}{36}$	(It is fine if the student converted 3 dozen to 36 [there are 12 in a dozen], although this was not expected.)
b. $\dfrac{5}{\$1}$	
c. $\dfrac{40 \text{ passengers}}{\text{train}}$ ***or*** $\dfrac{40 \text{ passengers}}{1 \text{ train}}$	
d. $\dfrac{1 \text{ gal}}{3 \text{ trips}}$	

2.

a. $\approx$ **4.374 yd**

$4\ \cancel{m}\left(\frac{1\text{ yd}}{0.914\ \cancel{m}}\right) \approx 4.374\text{ yd}$

b. $\approx$ **0.455 mi**

$800\ \cancel{yd}\left(\frac{1\text{ mi}}{1{,}760\ \cancel{yd}}\right) \approx 0.455\text{ mi}$

3. *Note:* It is fine if students used a different method to solve these problems; there are a variety of ways to find the missing value in a proportion.

a. $\boldsymbol{x = 8}$

$\frac{4}{10} = \frac{x}{20}; x = 8$	(multiplied $\frac{4}{10}$ by $\frac{2}{2}$ to finish forming an equivalent ratio with 20 in the denominator)

b. $\boldsymbol{x = 4}$

$\frac{2}{x} = \frac{10}{20}; x = 4$	(divided both the numerator and denominator of $\frac{10}{20}$ by 5 in order to form an equivalent ratio with 2 in the numerator)

c. $\boldsymbol{x = 4}$

$\frac{8}{22} = \frac{x}{11}; x = 4$	(divided both the numerator and denominator of $\frac{8}{22}$ by 2 in order to form an equivalent ratio with 11 in the denominator)

d. **300 mi**

$\frac{3\text{ days}}{100\text{ mi}} = \frac{9\text{ days}}{x}$	(wrote a proportion, using x to stand for the miles gone in 9 days)
$\frac{3\text{ days}}{100\text{ mi}}\left(\frac{3}{3}\right) = \frac{9\text{ days}}{300\text{ miles}}$	(multiplied by $\frac{3}{3}$ to form an equivalent fraction with 9 days in the numerator)

$x = 300$ mi; they could go 300 mi in 9 days.

Note: We knew to multiply by 3 because $9 \div 3 = 3$, so the numerator was multiplied by $\frac{3}{3}$ to go from 3 to 9.

e. **≈ 66.667 mi**	
$\frac{3 \text{ days}}{100 \text{ mi}} = \frac{2 \text{ days}}{x}$	(wrote a proportion, using *x* to stand for the miles gone in 9 days)
$\frac{3 \text{ days} \div 1.5}{100 \text{ mi} \div 1.5} \approx \frac{2 \text{ days}}{66.667 \text{ miles}}$	(divided both the numerator and denominator by 1.5 to form an equivalent fraction with 2 days in the numerator)

Note: We knew to divide by 1.5 because 3 ÷ 2 = 1.5, so the numerator was divided by 1.5 to go from 3 to 2.

4. $\mathbf{\frac{16}{75}}$

$$\frac{4(8)}{15(10)} = \frac{32}{150} = \frac{16}{75}$$

Worksheet 1.6

1.

a. $\mathbf{\frac{\$7}{bu}}$

b. $\mathbf{\frac{\$7}{3\ bu}}$

c. $\mathbf{\frac{9\ m}{3\ min}}$

2. $\mathbf{3\frac{m}{min}}$

$\frac{9 \text{ m}}{3 \text{ min}} = 3\frac{\text{m}}{\text{min}}$	(simplified the known numbers by dividing 9 by 3)

3. *Note:* Students were told not to round on 3a and 3b.

a. $\mathbf{3.28083989501\frac{yd}{s}}$

$\left(3\frac{\text{m}}{\text{s}}\right)\left(\frac{1 \text{ yd}}{0.9144 \text{ m}}\right) = \left(3\frac{\cancel{\text{m}}}{\text{s}}\right)\left(\frac{1 \text{ yd}}{0.9144 \cancel{\text{m}}}\right)$ $= 3.28083989501\frac{\text{yd}}{\text{s}}$	(multiplied by a conversion ratio worth 1; the meters canceled out)

b. $\mathbf{0.00186411357\ \frac{mi}{s}}$

$\left(3.28083989501\frac{\text{yd}}{\text{s}}\right)\left(\frac{1 \text{ mi}}{1{,}760 \text{ yd}}\right) =$ $\left(3.28083989501\frac{\cancel{\text{yd}}}{\text{s}}\right)\left(\frac{1 \text{ mi}}{1{,}760 \cancel{\text{yd}}}\right) = 0.00186411357\frac{\text{mi}}{\text{s}}$	(multiplied by a conversion ratio worth 1; the yards canceled out)

c. $\approx \mathbf{6.711\frac{mi}{hr}}$

$$\left(0.00186411357\frac{\text{mi}}{\text{s}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right) = \left(0.00186411357\frac{\text{mi}}{\cancel{\text{s}}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\text{ hr}}\right)$$

$\approx 6.711\frac{\text{mi}}{\text{hr}}$ (multiplied by conversion ratios worth 1; the seconds and minutes canceled out)

4.

a. $\mathbf{\frac{2\text{ m}}{\text{s}}}$

$$2\frac{\text{m}}{\text{s}} = \frac{2\text{ m}}{\text{s}}$$

b. $\mathbf{\frac{7\text{ ft}}{\text{s}}}$

$$7\frac{\text{ft}}{\text{s}} = \frac{7\text{ ft}}{\text{s}}$$

5. $\mathbf{\dfrac{8\frac{m}{s}}{6\frac{m}{s}}}$ (This can simplify to $\frac{4}{3}$ with units canceling, but students were told they did not need to simplify for this problem.)

6.

a. $\mathbf{2\frac{m}{s}}$

b. **16 ft**

$$8\frac{\text{ft}}{\cancel{\text{s}}}(2\ \cancel{\text{s}}) = 16\text{ ft}$$

c. **4**

$$\frac{\frac{8\text{ ft}}{\text{s}}}{\frac{2\text{ ft}}{\text{s}}} = \frac{8\ \cancel{\text{ft}}}{\cancel{\text{s}}}\left(\frac{\cancel{\text{s}}}{2\ \cancel{\text{ft}}}\right) = 4$$

7.

a. **250 lb**

$\frac{2.5 \text{ lb}}{1 \text{ gal}} = \frac{x}{100 \text{ gal}}$ (wrote a proportion)

$\frac{2.5 \text{ lb}}{1 \text{ gal}}\left(\frac{100}{100}\right) = \frac{250 \text{ lb}}{100 \text{ gal}}$ (multiplied by $\frac{100}{100}$ to form an equivalent fraction with 100 gal in the denominator)

$x = 250$ lb; 100 gallons of this milk will yield 250 lb of butter fat.

Note: We knew to multiply by $\frac{100}{100}$ because $100 \div 1 = 100$, so the denominator was multiplied by 100 to go from 1 to 100. Thus, to form an equivalent faction, we needed to multiply the numerator by that same amount.

b. **8.181° Centigrade**

Note: Students could have converted miles to meters instead. Just look to see if the answer is 8.181° Centigrade.

$295 \text{ m}\left(\frac{1 \cancel{\text{km}}}{1{,}000 \text{ m}}\right)\left(\frac{1 \text{ mi}}{1.609 \cancel{\text{km}}}\right) = 0.18334369173 \text{ mi}$ (converted from meters to miles)

$\frac{0.18334369173 \text{ mi}}{1° \text{ Centigrade}} = \frac{1.5 \text{ mi}}{x}$ (wrote a proportion)

$\frac{0.18334369173 \text{ mi}}{1° \text{ Centigrade}}\left(\frac{8.18135593238}{8.18135593238}\right) = \frac{1.5 \text{ mi}}{8.18135593238° \text{ Centigrade}}$ (multiplied by $\frac{8.18135593238}{8.18135593238}$ to form an equivalent fraction with 1.5 mi in the numerator)

Note: We knew to multiply by $\frac{8.18135593238}{8.18135593238}$ because $1.5 \div 0.18334369173 = 8.18135593238$, so the numerator was multiplied by 8.18135593238 to go from 0.18334369173 to 1.5.

$x \approx 8.181°$ Centigrade; 8.181° Centigrade is the approximate change in boiling point for an elevation of 1.5 mi.

Worksheet 1.7

1. *Note:* Again, note that students may arrange letters in different orders within a term. Both *xy* and *yx* mean the same thing.

a. $\mathbf{\frac{4c + 2b}{bc}}$

$\frac{4}{b}\left(\frac{c}{c}\right) = \frac{4c}{bc}$

$\frac{2}{c}\left(\frac{b}{b}\right) = \frac{2b}{bc}$ (multiplied each fraction by a fraction worth 1 to get a common denominator)

$\frac{4c}{bc} + \frac{2b}{bc} = \frac{4c + 2b}{bc}$ (added the numerators)

b. $\mathbf{\frac{xy + 6ay}{2ab}}$ ***or*** $\mathbf{\frac{6ay + xy}{2ab}}$	
$\frac{3y}{b}\left(\frac{2a}{2a}\right) = \frac{6ay}{2ab}$	(multiplied by a fraction worth 1 to get a common denominator)
$\frac{xy}{2ab} + \frac{6ay}{2ab} = \frac{xy + 6ay}{2ab}$ *or* $\frac{6ay + xy}{2ab}$	(added the numerators)
Note: If students already know how to factor, they might have written the answer as $\frac{(6a + x)y}{2ab}$, which is also valid.	
c. $\mathbf{\frac{bx + a}{b}}$ ***or*** $\mathbf{\frac{a + bx}{b}}$	
$x\left(\frac{b}{b}\right) + \frac{a}{b} = \frac{bx}{b} + \frac{a}{b} = \frac{bx + a}{b}$ *or* $\frac{a + bx}{b}$	(Since addition is commutative and order doesn't matter, we listed the *a* first in the final answer, as the convention is to list letters in alphabetical order.)
d. $\mathbf{423\frac{m}{min}}$	
$7\frac{\text{m}}{\text{s}}\left(\frac{60\text{ s}}{1\text{ min}}\right) = 7\frac{\text{m}}{\cancel{\text{s}}}\left(\frac{60\ \cancel{\text{s}}}{1\text{ min}}\right) = 420\frac{\text{m}}{\text{min}}$	(converted the $\frac{\text{m}}{\text{s}}$ to $\frac{\text{m}}{\text{min}}$ so as to have the answer in the specified units)
$420\frac{\text{m}}{\text{min}} + 3\frac{\text{m}}{\text{min}} = 423\frac{\text{m}}{\text{min}}$	(added the fractions)
e. $\mathbf{6.2\frac{in}{s}}$	
$6\frac{\cancel{\text{ft}}}{\cancel{\text{min}}}\left(\frac{1\ \cancel{\text{min}}}{60\text{ s}}\right)\left(\frac{12\text{ in}}{1\ \cancel{\text{ft}}}\right) = 1.2\frac{\text{in}}{\text{s}}$	(converted the $\frac{\text{ft}}{\text{min}}$ to $\frac{\text{in}}{\text{s}}$ so as to have the answer in the specified units)
$1.2\frac{\text{in}}{\text{s}} + 5\frac{\text{in}}{\text{s}} = 6.2\frac{\text{in}}{\text{s}}$	(added the terms)

2.

a. $\mathbf{=}$	(If $x = 0$, they would not be equal, as the left side would be undefined.)
b. $\mathbf{=}$	
c. $\neq$	

3.

a. $\approx \mathbf{295.276\frac{yd}{min}}$	
$\left(4.5\frac{\text{m}}{\text{s}}\right)\left(\frac{\text{yd}}{0.9144\text{ m}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \left(4.5\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1}{0.9144}\frac{\text{yd}}{\cancel{\text{m}}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\text{ min}}\right) \approx 295.276\frac{\text{yd}}{\text{min}}$	

	b. $\approx \mathbf{0.049 \dfrac{m}{s}}$	
	$\left(3.2\dfrac{\text{yd}}{\text{min}}\right)\left(\dfrac{0.9144\text{ m}}{1\text{ yd}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = \left(3.2\dfrac{\cancel{\text{yd}}}{\cancel{\text{min}}}\right)\left(\dfrac{0.9144\text{ m}}{1\ \cancel{\text{yd}}}\right)\left(\dfrac{1\ \cancel{\text{min}}}{60\text{ s}}\right) \approx 0.049\dfrac{\text{m}}{\text{s}}$	
4.	$\mathbf{\dfrac{1}{2}}$	
	$\dfrac{3\dfrac{\text{yd}}{\text{min}}}{\dfrac{6\text{ yd}}{\text{min}}} = \dfrac{3\ \cancel{\text{yd}}}{\cancel{\text{min}}}\left(\dfrac{\cancel{\text{min}}}{6\ \cancel{\text{yd}}}\right) = \dfrac{3}{6} = \dfrac{1}{2}$	(inverted and multiplied)
5.		
	a. ***Jack's Rate*** $= \mathbf{\dfrac{4\text{ problems}}{20\text{ min}}}$ ***Mike's Rate*** $= \mathbf{\dfrac{5\text{ problems}}{30\text{ min}}}$	
	b. ***Jack's Rate*** $= \mathbf{\dfrac{12\text{ problems}}{1\text{ hr}}}$ ***Mike's Rate*** $= \mathbf{\dfrac{10\text{ problems}}{1\text{ hr}}}$	
	Jack's Rate $= \dfrac{4\text{ problems}}{20\ \cancel{\text{min}}}\left(\dfrac{60\ \cancel{\text{min}}}{1\text{ hr}}\right) = \dfrac{12\text{ problems}}{1\text{ hr}}$ *Mike's Rate* $= \dfrac{5\text{ problems}}{30\ \cancel{\text{min}}}\left(\dfrac{60\ \cancel{\text{min}}}{1\text{ hr}}\right) = \dfrac{10\text{ problems}}{1\text{ hr}}$	
	c. **22 problems**	
	12 problems + 10 problems = 22 problems	
	They can solve 22 problems per hour combined.	
6.	**\$2**	
	$\dfrac{2}{3}(\$6) = \$4$	(computed the price of the discount)
	$\$6 - \$4 = \$2$	(found the amount left to pay after the discount)
	Students may also have solved like this:	
	$1 - \dfrac{2}{3} = \dfrac{3}{3} - \dfrac{2}{3} = \dfrac{1}{3}$	(calculated the portion of total to pay after the $\dfrac{2}{3}$ discount)
	$\dfrac{1}{3}(\$6) = \$2$	(calculated $\dfrac{1}{3}$ of the total)

Worksheet 1.8

Note: Remind students to also study for the quiz.

1.

a. $\boldsymbol{x}$	
$-(-(-(-x))) = x$	(An even number of negative signs results in a positive answer.)
b. $\boldsymbol{\frac{b}{x}}$	
$-\frac{b}{-x} = \frac{b}{x}$	(The opposite of the opposite is positive.)
c. $\mathbf{\frac{1}{2}}$	
$-\frac{-1}{2} = \frac{1}{2}$	(The opposite of the opposite is positive.)
d. $\boldsymbol{-abc}$	
$(-a)(-b)(-c) = -abc$	(An odd number of negative signs results in a negative answer.)
e. $\boldsymbol{\frac{5a+3}{2}}$	
$\frac{5a}{2} - \frac{-3}{2} = \frac{5a}{2} + \frac{3}{2} = \frac{5a+3}{2}$	(An even number of negative signs results in a positive answer.)
f. **–2**	
$5 - 7 = -2$	
g. **–5**	
$3 - 8 = -5$	

2.

a. **6**	
$-ax = -(-2)(3) = 6$	(An even number of negative signs results in a positive answer.)
b. $\boldsymbol{6 + -t}$	
c. **0**	(A value added to its opposite equals 0.)

3.

a. **–100 mi**	
$distance = speed(time)$	
$distance = \left(-50\,\frac{\text{mi}}{\cancel{\text{hr}}}\right)2\,\cancel{\text{hr}} = -100\text{ mi}$	

b. **$\approx 30.227\,\frac{\text{mi}}{\text{hr}}$ faster**	
$\frac{29\,\cancel{\text{ft}}}{\cancel{\text{s}}}\left(\frac{1\text{ mi}}{5{,}280\,\cancel{\text{ft}}}\right)\left(\frac{60\,\cancel{\text{s}}}{1\,\cancel{\text{min}}}\right)\left(\frac{60\,\cancel{\text{min}}}{1\text{ hr}}\right)$ $\approx 19.77272727\,\frac{\text{mi}}{\text{hr}}$	(converted $29\,\frac{\text{ft}}{\text{s}}$ to $\frac{\text{mi}}{\text{hr}}$, as students were asked to compare the speeds in miles per hour)
$50\,\frac{\text{mi}}{\text{hr}} - 19.77272727\,\frac{\text{mi}}{\text{hr}} \approx 30.227\,\frac{\text{mi}}{\text{hr}}$	(compared $50\,\frac{\text{mi}}{\text{hr}}$ with $19.773272727\,\frac{\text{mi}}{\text{hr}}$)
c. **$20\,\frac{\text{J}}{\text{s}}$ *or* 20 W**	
$P = VI$	
$P = \left(-4\,\frac{\text{J}}{\cancel{€}}\right)\left(-5\,\frac{\cancel{€}}{\text{s}}\right) = 20\,\frac{\text{J}}{\text{s}}$ *or* 20 W	(W stands for watts, which is a $\frac{\text{J}}{\text{s}}$.)
d. **−$360**	
$\frac{-\$30}{1\text{ month}} = \frac{x}{12\text{ months}}$	(wrote a proportion; notice that we used 12 months to represent 1 year so as to keep the units consistent)
$\frac{-\$30}{1\text{ month}}\left(\frac{12}{12}\right) = \frac{-\$360}{12\text{ months}}$	(multiplied by $\frac{12}{12}$ to form a proportion with 12 months in the denominator)
$x = -\$360$; $360 is the amount owed for the entire year.	

4.

a. **$\frac{2xb - 3ay}{by}$**	
$\frac{2x}{y} + \frac{-3a}{b} = \frac{2x}{y}\left(\frac{b}{b}\right) + \frac{-3a}{b}\left(\frac{y}{y}\right) = \frac{2xb}{by} + \frac{-3ay}{by} = \frac{2xb - 3ay}{by}$	
b. **−5**	
$\frac{5a}{3}\left(\frac{-6}{2a}\right) = \frac{-30a}{6a} = -5$	(The *a* in the numerator and denominator cancel out, and –30 divided by 6 is –5.)
c. **10 hr**	
$\frac{-50\text{ in}}{\frac{-5\text{ in}}{\text{hr}}} = -50\,\cancel{\text{in}}\left(-\frac{\text{hr}}{5\,\cancel{\text{in}}}\right) = 10\text{ hr}$	(An even number of negative signs results in a positive answer and 50 divided by 5 is 10.)

5. *Note:* Remind students if needed that conversion ratios are in Appendix B.

a. **$\approx 7{,}874.016\,\frac{\text{ft}}{\text{min}}$**	
$40\,\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\left(\frac{60\,\cancel{\text{s}}}{1\text{ min}}\right)\left(\frac{100\,\cancel{\text{cm}}}{1\,\cancel{\text{m}}}\right)\left(\frac{1\text{ ft}}{30.48\,\cancel{\text{cm}}}\right) \approx 7{,}874.016\,\frac{\text{ft}}{\text{min}}$	

b. $\approx$ **5,955.512 $\frac{\textbf{ft}}{\textbf{min}}$**

$$30\frac{\cancel{m}}{\cancel{s}}\left(\frac{60\ \cancel{s}}{1\text{ min}}\right)\left(\frac{100\ \cancel{cm}}{1\ \cancel{m}}\right)\left(\frac{1\text{ ft}}{30.48\ \cancel{cm}}\right) + 50\,\frac{\text{ft}}{\text{min}} = 5{,}905.511811\frac{\text{ft}}{\text{min}} + 50\,\frac{\text{ft}}{\text{min}}$$

$$\approx 5{,}955.512\frac{\text{ft}}{\text{min}}$$

6.

a. **8*axz***

b. **The Commutative and Associative Properties**

c. **2; if $x = 2$, that would have us dividing by 0 and division by 0 is not possible, as you can't divide something by nothing.**

Chapter 2 Exploring Exponents

Worksheet 2.1

1.

a. $\mathbf{4^4}$	
b. $\mathbf{-4^3}$ ***or*** $\mathbf{(-4)^3}$	(3 negative signs will result in an odd answer so -4^3 means the same thing as $(-4)^3$.)
c. $\boldsymbol{x^6}$	
d. $\boldsymbol{a^2}$ ***or*** $\boldsymbol{(-a)^2}$	(The 2 negative signs cancel each other out, so a^2 means the same thing as $(-a)^2$.)
e. $\boldsymbol{y^3x^2}$	
f. $\boldsymbol{x^1}$	(By definition, $x^1 = x$.)

2.

a. **490**	
$(2^2 + 3 \times 1)^2 \bullet 10 = (4 + 3)^2 \bullet 10 = (7)^2 \bullet 10 = 49 \bullet 10 = 490$	
b. **–1**	
$(2 - 1)^3 - 2 = (1)^3 - 2 = 1 - 2 = -1$	

3.

a. $\boldsymbol{-a^3}$	
$-a(-a)(-a) = -a^3$	
b. **16**	
$(-x)^4 = (-2)^4 = (-2)(-2)(-2)(-2) = 16$	(An even number of negative signs yields a positive answer.)
c. **–16**	
$-(x)^4 = -(2)^4 = -(2)(2)(2)(2) = -16$	(The negative sign was outside of the parentheses this time, so we had to simplify inside the parentheses first, and *then* take the opposite of that.)
d. **–4**	
$-(-2)^2 = -(-2)(-2) = -(4) = -4$	
e. **4**	
$(-(-2))^2 = (2)^2 = (2)(2) = 4$	

4.

a. **25**	
$(4 + 3 - 2)^2 = (7 - 2)^2 = (5)^2 = 25$	
b. **36**	
$\left(\frac{3^2 + 7(3)}{5}\right)^2 = \left(\frac{9 + 21}{5}\right)^2 = \left(\frac{30}{5}\right)^2 = (6)^2 = 36$	

5.

a. $\mathbf{\frac{8}{y}}$	
$\left(\frac{2x}{y}\right)\left(\frac{4}{x}\right) = \left(\frac{2x}{y}\right)\left(\frac{4}{x}\right) = \frac{8}{y}$	
b. $\mathbf{\frac{6a-2b}{ab}}$	
$\frac{6}{b} + \frac{-2}{a} = \frac{6}{b}\left(\frac{a}{a}\right) + \frac{-2}{a}\left(\frac{b}{b}\right) = \frac{6a}{ab} + \frac{-2b}{ab} = \frac{6a-2b}{ab}$	
c. $\mathbf{-85\frac{yd}{min}}$	
$5\frac{\text{ft}}{\text{s}}\left(\frac{1\text{ yd}}{3\text{ ft}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 5\frac{\text{ft}}{\text{s}}\left(\frac{1\text{ yd}}{3\text{ ft}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)$ $= 100\frac{\text{yd}}{\text{min}}$	(converted to have a common denominator; was told to give answer in $\frac{\text{yd}}{\text{min}}$)
$15\frac{\text{yd}}{\text{min}} - 100\frac{\text{yd}}{\text{min}} = -85\frac{\text{yd}}{\text{min}}$	(subtracted the fractions)
d. $\mathbf{\frac{1}{4}}$	
$\frac{98}{392} = \frac{2 \bullet 7 \bullet 7}{2 \bullet 2 \bullet 2 \bullet 7 \bullet 7} = \frac{1}{4}$	

6.

a. $\mathbf{-6\frac{J}{s}}$ ***or*** **−6 watts**	
$P = \left(-3\frac{\text{J}}{\text{C}}\right)\left(2\frac{\text{C}}{\text{s}}\right) = -6\frac{\text{J}}{\text{s}}$ *or* −6 watts	
b. ***Volume*** **≈ 0.039 m³**	
Finding the radius:	
$r = \frac{1}{2}d = \frac{1}{2}0.25\text{ m} = 0.125\text{ m}$	(formula from Appendix B; inserted values and simplified)
Finding the volume:	
$V = \pi r^2 h$	(formula from Appendix B)
$V = \pi\,(0.125\text{ m})^2(0.8\text{ m}) \approx 0.039\text{ m}^3$	(inserted values and simplified)

Worksheet 2.2

1. *Note:* Students were instructed to use a negative exponent.

a. $\mathbf{4^{-5}}$

b. $\mathbf{4^{-1}}$

c. $\boldsymbol{x^{-1}}$

d. $\mathbf{m \cdot s^{-1}}$ ***or*** $\mathbf{m\ s^{-1}}$

e. $\boldsymbol{x^{-3}}$

2. *Note:* Students were instructed to use a positive exponent.

a. $\boldsymbol{x^4}$

$$\frac{1}{x^{-4}} = x^4$$

b. $\boldsymbol{-x^4}$

$$\frac{1}{-x^{-4}} = -x^4$$

c. $\boldsymbol{\frac{1}{x^9}}$

$$x^{-9} = \frac{1}{x^9}$$

d. $\boldsymbol{\frac{1}{(-b)^3}}$ ***or*** $\boldsymbol{\frac{-1}{b^3}}$

$$\frac{1}{-b}\left(\frac{1}{-b}\right)\left(\frac{1}{-b}\right) = \frac{1}{(-b)^3}$$

(We put the negative sign inside the parentheses to show that it's the *opposite of b* we have to multiply 3 times.)

e. $\boldsymbol{\frac{y^3}{x^3}}$

$$-\frac{yyy}{-xxx} = \frac{y^3}{x^3}$$

(The negative signs canceled out.)

3. $\mathbf{\frac{Hz}{V}}$

Note: Students were instructed to not use a negative exponent.

4. *Note:* Students were instructed not to round.

a. $\mathbf{6.254 \times 10^{10}\ km^3}$

b. $\mathbf{1.2 \times 10^{-12}\ kg \cdot mol^{-1}}$

(The unit of measure could also be written $\frac{\text{kg}}{\text{mol}}$.)

5. *Note:* Students were instructed not to round.

a. **484,000,000,000,000** $\mathbf{Hz \cdot V^{-1}}$

(The unit of measure could also be written $\frac{\text{Hz}}{\text{V}}$.)

b. **0.000000000100001495 m**

c. **62,000,000 $\Omega^{-1} \cdot m^{-1}$**	(The unit of measure could also be written $\frac{1}{\Omega \cdot m}$.)

6. **2.5×10^{-15} m²**

$$\left(\frac{1}{4}\right)(1 \times 10^{-14}\ \text{m}^2) = 0.25 \times 10^{-14}\ \text{m}^2 = 2.5 \times 10^{-15}\ \text{m}^2$$

7.

a. $\mathbf{\frac{5ax + 4c}{2cx}}$

$$\frac{5a}{2c} + \frac{2}{x} = \left(\frac{5a}{2c}\right)\left(\frac{x}{x}\right) + \left(\frac{2}{x}\right)\left(\frac{2c}{2c}\right) = \frac{5ax}{2cx} + \frac{4c}{2cx} = \frac{5ax + 4c}{2cx}$$

b. $\mathbf{-3\frac{ft}{s}}$

$$2\frac{\text{ft}}{\text{s}} - 60\frac{\text{in}}{\text{s}} = 2\frac{\text{ft}}{\text{s}} - 60\frac{\cancel{\text{in}}}{\text{s}}\left(\frac{1\ \text{ft}}{12\ \cancel{\text{in}}}\right) = 2\frac{\text{ft}}{\text{s}} - 5\frac{\text{ft}}{\text{s}} = -3\frac{\text{ft}}{\text{s}}$$

Worksheet 2.3A

1.

a. ***d***

$d^5d^{-4} = d^{5-4} = d^1 = d$

b. $\mathbf{y^{-3}}$ ***or*** $\mathbf{\frac{1}{y^3}}$

$y^5yy^{-9} = y^5y^1y^{-9} = y^{5+1-9} = y^{-3}$ *or* $\frac{1}{y^3}$

c. $\mathbf{a^4}$

$\frac{a^8}{a^4} = a^{8-4} = a^4$

d. $\mathbf{a^{12}}$

$\frac{a^8}{a^{-4}} = a^8a^4 = a^{12}$

e. **4*b***

$\frac{-4ab^2}{-ab} = \frac{4ab^2}{ab} = \frac{4b^2}{b} = 4b^2b^{-1} = 4b^{2-1} = 4b^1 = 4b$

f. $\mathbf{5x^{11}}$

$5x^{2--9} = 5x^{2+9} = 5x^{11}$

Worksheet 2.3B

1.

a.	(Only if y equals 2 are these equal; the first equation equals y^6.)
b. $=$	
c. $=$	(The a^2 cancels out.)
d.	(These do not equal. In the numerator, the parentheses mean we're taking $(-a)(-a)$. So the numerator would simplify to a^2, while the denominator would be $-a^2$, simplifying to -1, not 1.)
e. $=$	(The denominators are the same, as $3^2 = 9$.)

2. *Note:* Students only need to write the correct sign; we show how the expressions simplified here for clarity.

a. $<$	
$\frac{1}{64} < 216$	
b. $>$	
$5 > 4$	
c. $=$	
$3^4 = 3^4$	($\frac{3^6}{3^2}$ simplifies to 3^4, as $\frac{3^6}{3^2} = 3^{6-2} = 3^4$.)
d. $=$	
$3^4 = 3^4$	(The denominator equals $(-3)(-3)$, which can be thought of as $(-1)(3)(-1)(3)$. The -1s cancel out, leaving us with $(3)(3)$, or 3^2.)
e. $<$	
$-3^4 < 3^4$	(Here the negative sign was not part of what was being squared, so it made the whole fraction negative.)

3.

a. $\boldsymbol{y^4}$	
$\frac{y^6}{y^2} = y^{6-2} = y^4$	
b. $\boldsymbol{3x^{-7}}$ ***or*** $\boldsymbol{\frac{3}{x^7}}$	
$\frac{3x^{-1}}{x^6} = 3x^{-1-6} = 3x^{-7}$ *or* $\frac{3}{x^7}$	
c. $\boldsymbol{-10a^{-1}b^2}$ ***or*** $\boldsymbol{\frac{-10b^2}{a}}$	
$-5ab^2(2a^{-2}) = -10a^{1+-2}b^2 = -10a^{-1}b^2$ *or* $\frac{-10b^2}{a}$	

4. **The order of operations says to multiply first and then add.** We can't add $5ab^2 + 2a^{-2}$ until we complete the multiplication of 5 times a times b^2 and 2 times a^{-2}. In $5ab^2(2a^{-2})$, we are dealing with only multiplication, which is commutative and can be done in any order. The commutative property we're following describes the consistent way God governs all things, and the order of operations describes an agreed-upon convention for writing operations.

5.

a.	$\mathbf{2.0 \times 10^{-17}}$	
	20×10^{-18}	(multiplied 5×4, and $10^{-12} \times 10^{-6}$)
	2.0×10^{-17}	(rewrote in scientific notation)
b.	$\mathbf{6 \times 10^{38}}$	(multiplied 3×2 and $10^{32} \times 10^{6}$)

6. *Note:* Students were told to use a negative exponent.

a.	$\boldsymbol{x = 3 \times 10^8}$ **transistors**; a 2 mm^2 chip would have 3×10^8 transistors if the ratio were the same.	
	$\dfrac{1.5 \times 10^8 \text{ transistors}}{1 \text{ mm}^2} = \dfrac{x}{2 \text{ mm}^2}$	(set up a ratio)
	$\dfrac{1.5 \times 10^8 \text{ transistors}}{1 \text{ mm}^2}\left(\dfrac{2}{2}\right) = \dfrac{3 \times 10^8 \text{ transistors}}{2 \text{ mm}^2}$	(multiplied by $\dfrac{2}{2}$)
	$x = 3 \times 10^8$ transistors; a 2 mm^2 chip would have 3×10^8 transistors if the ratio were the same.	
b.	$\boldsymbol{F = kq_1q_2r^{-2}}$	

Worksheet 2.4

1.

a.	$\boldsymbol{8a^3c^9}$	
	$(2ac^3)^3 = 2^3a^3c^{3(3)} = 8a^3c^9$	
b.	$\boldsymbol{\dfrac{1}{x^4}}$ ***or*** $\boldsymbol{x^{-4}}$	
	$\left(\dfrac{1}{x^2}\right)^2 = \dfrac{1^{1(2)}}{x^{2(2)}} = \dfrac{1}{x^4}$	
c.	$\boldsymbol{m^{-12}}$ ***or*** $\boldsymbol{\dfrac{1}{m^{12}}}$	
	$(m^{-6})^2 = m^{-6(2)} = m^{-12}$	
d.	$\boldsymbol{m^{-14}}$ ***or*** $\boldsymbol{\dfrac{1}{m^{14}}}$	
	$\dfrac{(m^{-6})^2}{m^2} = \dfrac{m^{-6(2)}}{m^2} = \dfrac{m^{-12}}{m^2} = m^{-12-2} = m^{-14}$	(Since the order of operations says to solve exponents and roots before multiplication or division, we had to square the m^6 before dividing by m^2.)

2.

a. **64**

$(4)^3 = 4^3 = 64$

b. **64** — *Note:* Students were instructed to show their work.

$V = (2^2)^3 = (4)^3 = 64$ (simplified inside the parentheses)

c. $\boldsymbol{b^6}$

$V = (b^2)^3 = b^{2(3)} = b^6$

d. **64**

$V = b^6 = 2^6 = 64$ (2^6 equals 64)

3.

a. **32,768**

$2^{-5(-3)} = 2^{15} = 32{,}768$

b. $\boldsymbol{256b^2}$

$$\frac{4^4 b^{2(4)}}{b^6} = \frac{4^4 b^8}{b^6} = 4^4 b^{8-6} = 4^4 b^2 = 256b^2$$

c. $\boldsymbol{\frac{1}{x^{-6}}}$ ***or*** $\boldsymbol{x^6}$

$$\left(\frac{1}{x^{-2}}\right)^3 = \frac{1}{x^{-2(3)}} = \frac{1}{x^{-6}} \text{ or } x^6$$

Note: Students may also have solved like this: $\left(\frac{1}{x^{-2}}\right)^3 = \left(\frac{1}{x^{-2}}\right)\left(\frac{1}{x^{-2}}\right)\left(\frac{1}{x^{-2}}\right) = \frac{1}{x^{-6}}$ *or* x^6

4. $\boldsymbol{(2aaa)(2aaa)}$ ***or*** $\boldsymbol{4aaaaaa}$

$(2a^3)^2 = (2a^3)(2a^3) = (2aaa)(2aaa)$

5.

a. $\boldsymbol{y^2}$

$$\frac{(-y^4)}{y^2} = \frac{y^4}{y^2} = y^{4-2} = y^2$$

b. $\boldsymbol{-y^2}$

$$\frac{-y^4}{y^2} = -y^{4-2} = -y^2$$

c. $\boldsymbol{\frac{1}{x^{13}}}$ ***or*** $\boldsymbol{x^{-13}}$

$$-\frac{-(x^3)^{-3}}{x^4} = \frac{x^{-9}}{x^4} = \frac{1}{x^{13}} \text{ or } x^{-13}$$

6.

a. $\boldsymbol{6.0 \times 10^{-3}}$

$(2.0 \times 10^{-2})(3.0 \times 10^{-1}) = (2.0)(3.0) \times 10^{-2+-1} = 6.0 \times 10^{-3}$

b. $\boldsymbol{4.2 \times 10^{32}}$

$(7.0 \times 10^{30})(6.0 \times 10^{1}) = (7.0)(6.0) \times 10^{30+1} = 42.0 \times 10^{31} = 4.2 \times 10^{32}$

7.

a. $\boldsymbol{x^3}$

$x^2x = x^2x^1 = x^{2+1} = x^3$

b. **2**

$\dfrac{x^2x}{\frac{x^3}{2}} = x^2x\left(\dfrac{2}{x^3}\right) = \dfrac{2x^2x^1}{x^3} = \dfrac{2x^{2+1}}{x^3}$ (Any number divided by itself equals 1, so x^3 divided by itself equals 1.)

$= \dfrac{2x^3}{x^3} = 2(1) = 2$

8. **≈ 1.309 in³**

Finding the radius:

$r = \frac{1}{2}d = \frac{1}{2}(1 \text{ in}) = 0.5 \text{ in}$

Calculating the volume:

$V = \pi r^2 \frac{h}{3} = \pi(0.5 \text{ in})^2\left(\frac{5 \text{ in}}{3}\right) \approx 1.309 \text{ in}^3$ (inserted values into formula and simplified)

Note: The $(0.5 \text{ in})^2$ simplifies to 0.25 in^2, which then gets multiplied by 5 in, giving an ending unit of in^3.

Worksheet 2.5A

1. *Note:* It's not necessary for students to write out in • in; we are showing it for clarity.

a. **25.806 cm²**

$4 \text{ in}^2 = 4 \cancel{\text{in}} \cdot \cancel{\text{in}}\left(\frac{2.54 \text{ cm}}{1 \cancel{\text{in}}}\right)\left(\frac{2.54 \text{ cm}}{1 \cancel{\text{in}}}\right) \approx 25.806 \text{ cm} \cdot \text{cm} = 25.806 \text{ cm}^2$

b. **0.0579 ft³**

$100 \text{ in}^3 = 100 \cancel{\text{in}} \cdot \cancel{\text{in}} \cdot \cancel{\text{in}}\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right)\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right)\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right) \approx 0.0579 \text{ ft} \cdot \text{ft} \cdot \text{ft} = 0.0579 \text{ ft}^3$

2.

a. $\boldsymbol{8\frac{\text{ft}^3}{\text{s}^6}}$ ***or*** $\boldsymbol{8 \text{ ft}^3 \text{s}^{-6}}$ ***or*** $\boldsymbol{\frac{8 \text{ ft}^3}{\text{s}^6}}$

$\left(2\frac{\text{ft}}{\text{s}^2}\right)^3 = 2^3\frac{\text{ft}^3}{\text{s}^{2(3)}} = 8\frac{\text{ft}^3}{\text{s}^6}$ *or* $8 \text{ ft}^3 \text{s}^{-6}$ *or* $\frac{8 \text{ ft}^3}{\text{s}^6}$

b. $\boldsymbol{-4\frac{\text{ft}^2}{\text{s}^6}}$

$\dfrac{\left(2\frac{\text{ft}}{\text{s}^2}\right)^3}{-2 \text{ ft}} = \dfrac{8\frac{\text{ft}^3}{\text{s}^6}}{-2 \text{ ft}} = \dfrac{8\frac{\text{ft}^3}{\text{s}^6}}{\frac{-2 \text{ ft}}{1}} = 8\frac{\text{ft}^3}{\text{s}^6}\left(\frac{1}{-2 \text{ ft}}\right) = \dfrac{8 \text{ ft}^3}{-2 \text{ ft} \cdot \text{s}^6} = \dfrac{-4 \text{ ft}^3}{\text{ft} \cdot \text{s}^6} = \dfrac{-4 \text{ ft}^3}{\text{ft}^1 \cdot \text{s}^6} =$

$\dfrac{-4 \text{ ft}^{3-1}}{\text{s}^6} = -4\dfrac{\text{ft}^2}{\text{s}^6}$

c. $\mathbf{81\frac{mi^4}{hr^8}}$ ***or*** $\mathbf{81\ mi^4\ hr^{-8}}$ ***or*** $\mathbf{\frac{81\ mi^4}{hr^8}}$	

$$\left(3\frac{\text{mi}}{\text{hr}^2}\right)^4 = 3^4\frac{\text{mi}^4}{\text{hr}^{2(4)}} = 81\frac{\text{mi}^4}{\text{hr}^8} \textit{ or } 81\ \text{mi}^4\ \text{hr}^{-8} \textit{ or } \frac{81\ \text{mi}^4}{\text{hr}^8}$$

Worksheet 2.5B

1.

a. $\mathbf{\frac{m^3}{K}}$	

$$\frac{\frac{\text{J}}{\text{K}}}{\frac{\text{J}}{\text{m}^3}} = \frac{\cancel{\text{J}}}{\text{K}}\left(\frac{\text{m}^3}{\cancel{\text{J}}}\right) = \frac{\text{m}^3}{\text{K}}$$

b. $\mathbf{kg \bullet m^{-1}}$ ***or*** $\mathbf{\frac{kg}{m}}$	

$$\frac{\text{kg}\frac{\text{m}}{\text{s}^2}}{\frac{\text{m}^2}{\text{s}^2}} = \frac{\frac{\text{kg} \bullet \text{m}}{\text{s}^2}}{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{kg} \bullet \text{m}}{\cancel{\text{s}^2}}\left(\frac{\cancel{\text{s}^2}}{\text{m}^2}\right) = \frac{\text{kg} \bullet \text{m}^1}{\text{m}^2} = \text{kg} \bullet \text{m}^{1-2} = \text{kg} \bullet \text{m}^{-1} \textit{ or } \frac{\text{kg}}{\text{m}}$$

2.

a. $\mathbf{42\ kg \bullet \frac{m^2}{s^3}}$	

$P = Fv$

$$P = \left(7\ \text{kg}\frac{\text{m}}{\text{s}^2}\right)\left(6\frac{\text{m}}{\text{s}}\right) = 7(6)\text{kg} \bullet \frac{\text{mm}}{\text{s}^2\text{s}} = 42\ \text{kg} \bullet \frac{\text{m}^1\text{m}^1}{\text{s}^2\text{s}^1} = 42\ \text{kg} \bullet \frac{\text{m}^{1+1}}{\text{s}^{2+1}} = 42\ \text{kg} \bullet \frac{\text{m}^2}{\text{s}^3}$$

b. **42 W or 42 watts**	
c. $\mathbf{48\ kg \bullet \frac{m^2}{s^3}}$ ***or*** **48 W** ***or*** **48 watts**	

$P = Fv$

$$P = \left(6\ \text{kg}\frac{\text{m}}{\text{s}^2}\right)\left(8\frac{\text{m}}{\text{s}}\right) = 6(8)\text{kg} \bullet \frac{\text{mm}}{\text{s}^2\text{s}} = 48\ \text{kg} \bullet \frac{\text{m}^1\text{m}^1}{\text{s}^2\text{s}^1} = 48\ \text{kg} \bullet \frac{\text{m}^{1+1}}{\text{s}^{2+1}} = 48\ \text{kg} \bullet \frac{\text{m}^2}{\text{s}^3} \textit{ or}$$

48 W *or* 48 watts

d. $\mathbf{kg \cdot m^2 s^{-3}}$	*Note:* Students were instructed to use a negative exponent to show s^3.

3.

a. $\mathbf{2.25\frac{kg^2}{m^2}}$	

$$\frac{45\ \text{kg}(2{,}000\ \text{kg})}{(200\ \text{m})^2} = \frac{45(2{,}000)\text{kg} \bullet \text{kg}}{200^2\text{m}^2} = \frac{90{,}000\ \text{kg}^2}{40{,}000\ \text{m}^2} = 2.25\frac{\text{kg}^2}{\text{m}^2}$$

b. $\approx \mathbf{1.501 \times 10^{-10} \frac{kg^2}{m^2}}$	*Note:* Students were instructed to give answer in scientific notation.
$(6.67 \times 10^{-11})\left(2.25\frac{kg^2}{m^2}\right) \approx 15.008 \times 10^{-11}\frac{kg^2}{m^2}$	(completed the multiplication)
$\approx 1.501 \times 10^{-10}\frac{kg^2}{m^2}$	(rewrote in scientific notation per instructions)
c. $\mathbf{1.501 \times 10^{-10}\ kg \cdot \frac{m}{s^2}}$	*Note:* Students were instructed to show how they obtained the correct units. Not every step needs written out — just enough to see that they did the work to see what the units should be.

$$(1.501 \times 10^{-10})\left(\frac{kg^2}{m^2}\right)\left(\frac{m^3}{kg \cdot s^2}\right) = (1.501 \times 10^{-10})\left(\frac{kg^2 \cdot m^3}{m^2 \cdot kg \cdot s^2}\right)$$

$$= (1.501 \times 10^{-10})\left(\frac{kg^2 \cdot m^3}{m^2 \cdot kg^1 \cdot s^2}\right) = (1.501 \times 10^{-10})\left(\frac{kg^{2-1} \cdot m^{3-2}}{s^2}\right)$$

$$= (1.501 \times 10^{-10})\left(\frac{m^1 \cdot kg^1}{s^2}\right) = \left(1.501 \times 10^{-10}\right)\left(\frac{m \cdot kg}{s^2}\right) = 1.501 \times 10^{-10}\ kg \cdot \frac{m}{s^2}$$

d. **$\mathbf{1.501 \times 10^{-10}}$ N *or* $\mathbf{1.501 \times 10^{-10}}$ newtons**

Worksheet 2.6A

1.

a. $\mathbf{\pm 16^{\frac{1}{2}}}$	
$\pm\sqrt{16} = \pm 16^{\frac{1}{2}}$	(Notice that the ± sign was included before the root sign, signifying that we're looking for either the positive *or* the negative square root; we needed to keep that sign to specify this in fractional exponential notation as well, as both notations mean the positive root without the ± in front.)
b. $\mathbf{55^{\frac{1}{4}}}$	
$\sqrt[4]{55} = 55^{\frac{1}{4}}$	
c. $\mathbf{17^{\frac{1}{3}}}$	
$\sqrt[3]{17} = 17^{\frac{1}{3}}$	
d. $\mathbf{30^{\frac{1}{5}}}$	
$\sqrt[5]{30} = 30^{\frac{1}{5}}$	
e. $\mathbf{36^{\frac{1}{2}}}$	
$\sqrt{36} = 36^{\frac{1}{2}}$	

f. $\pm\mathbf{81^{\frac{1}{4}}}$	
$\pm\sqrt[4]{81} = \pm 81^{\frac{1}{4}}$	(The ± sign lets us know we care about the negative root too; we need to include that when rewriting in fractional notation, as the notation alone is defined as meaning the positive root.)

2.

a. $\pm\mathbf{4}$	
$\pm\sqrt{16} = \pm 4$	
b. $\approx \mathbf{2.723}$	
$\sqrt[4]{55} \approx 2.723$	(There is not a ± sign in front, so the notation is defined as the positive root.)
c. $\approx \mathbf{2.571}$	
$\sqrt[3]{17} \approx 2.571$	
d. $\approx \mathbf{1.974}$	
$\sqrt[5]{30} \approx 1.974$	
e. $\mathbf{6}$	
$36^{\frac{1}{2}} = 6$	(There is not a ± sign in front, so the notation is defined as the positive root.)
f. $\pm\mathbf{3}$	
$= \pm 81^{\frac{1}{4}} = \pm 3$	

3.

a. $\mathbf{\frac{1}{\sqrt[4]{a}}}$	
$\frac{1}{a^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{a}}$	
b. $\mathbf{\frac{3}{5\sqrt{x}}}$	
$\frac{3}{5x^{\frac{1}{2}}} = \frac{3}{5\sqrt{x}}$	

Worksheet 2.6B

1.

a. **±8**

$\pm 64^{\frac{1}{2}} = \pm\sqrt{64} = \pm 8$

b. **≈ 3.162**

$\sqrt{4+6} = \sqrt{10} \approx 3.162$

c. **3**

$(4+5+234)^{\frac{1}{5}} = (243)^{\frac{1}{5}} = \sqrt[5]{243} = 3$

2.

a. **Either** (A square root is an even root, as 2 is an even number, so the answer could be either positive or negative.)

b. **Positive** (A cubed root is an odd root, as 3 is an odd number, so the answer has to be positive, since 15 is a positive number and the cubed root of a negative number would give a negative answer.)

c. **Either** (A fourth root is an even root, as 4 is an even number, so the answer could be either positive or negative.)

3.

a. **3**

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

b. $\mathbf{\frac{1}{3}}$

$27^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

Note: Students were instructed to give their answer as a fraction in order to make sure they did not use a calculator and really understand the fractional notation.

4.

a. $\boldsymbol{A^{\frac{1}{2}}}$

$s = A^{\frac{1}{2}}$

b. $\boldsymbol{V^{\frac{1}{3}}}$

$s = V^{\frac{1}{3}}$

c. $\approx$ **1.551 in**

Calculating the radius needed:

$r = \left(\frac{V}{\pi h}\right)^{\frac{1}{2}}$	(formula given)
$r = \left(\frac{45.36\text{ in}^3}{\pi(6\text{ in})}\right)^{\frac{1}{2}} = (2.40642274\text{ in})^{\frac{1}{2}} \approx 1.551\text{ in}$	(substituted values and calculated on a calculator)

5.

a. $\boldsymbol{y^{\frac{1}{2}}}$

$\sqrt{y} = y^{\frac{1}{2}}$

b. $\boldsymbol{\frac{1}{A^{\frac{1}{2}}}}$

$\frac{1}{\sqrt{A}} = \frac{1}{A^{\frac{1}{2}}}$

c. $\boldsymbol{\frac{1}{V^{\frac{1}{3}}}}$

$\frac{1}{\sqrt[3]{V}} = \frac{1}{V^{\frac{1}{3}}}$

6.

a. $\mathbf{1.2 \times 10^{-6}}$

$(6.0 \times 10^8)(2.0 \times 10^{-15}) = (6.0)(2.0) \times 10^{8+-15} = 12.0 \times 10^{-7} = 1.2 \times 10^{-6}$

b. $\mathbf{2.0 \times 10^{-5}}$

$(4.0 \times 10^{-6})(5.0 \times 10^0) = (4.0)(5.0) \times 10^{-6+0} = 20.0 \times 10^{-6} = 2.0 \times 10^{-5}$

7.

a. $\mathbf{\frac{16\ ft^4}{s^2}}$

$\left(4\frac{\text{ft}^2}{\text{s}}\right)^2 = \frac{4^2\text{ft}^{2(2)}}{\text{s}^2} = \frac{16\ \text{ft}^4}{\text{s}^2}$

b. $\mathbf{5\frac{m^2}{s^2}}$

$\frac{\left(5\frac{\text{m}^2}{\text{s}}\right)^2}{5\text{ m}^2} = \frac{\frac{25\text{ m}^4}{\text{s}^2}}{5\text{ m}^2} = \frac{25\text{ m}^4}{\text{s}^2}\left(\frac{1}{5\text{ m}^2}\right) = \frac{25\text{ m}^4}{5\text{ m}^2 \bullet \text{s}^2} = 5\frac{\text{m}^4}{\text{m}^2 \bullet \text{s}^2} = 5\frac{\text{m}^{4-2}}{\text{s}^2} = 5\frac{\text{m}^2}{\text{s}^2}$

Worksheet 2.7A

1.

a. $\boldsymbol{a^{\frac{11}{3}}}$

$a^{\frac{1}{3}}a^{\frac{4}{3}}a^2 = a^{\frac{1}{3}+\frac{4}{3}+\frac{6}{3}} = a^{\frac{11}{3}}$

b. $\boldsymbol{a^{\frac{3}{2}}}$

$\left(a^{\frac{3}{4}}\right)^2 = a^{\left(\frac{3}{4}\right)(2)} = a^{\frac{3(2)}{4}} = a^{\frac{6}{4}} = a^{\frac{3}{2}}$

c. $\boldsymbol{a^{\frac{3}{2}}}$

$(a^2)^{\frac{3}{4}} = a^{2\left(\frac{3}{4}\right)} = a^{\frac{2(3)}{4}} = a^{\frac{6}{4}} = a^{\frac{3}{2}}$

d. $\boldsymbol{b^{\frac{19}{4}}}$

$b^{\frac{3}{4}}b^4 = b^{\frac{3}{4}+4} = b^{\frac{3}{4}+\frac{16}{4}} = b^{\frac{19}{4}}$

e. $\boldsymbol{b^{\frac{13}{4}}}$

$b^{-\frac{3}{4}}b^4 = b^{-\frac{3}{4}+4} = b^{-\frac{3}{4}+\frac{16}{4}} = b^{\frac{13}{4}}$

f. $\boldsymbol{d^{\frac{3}{2}}}$

$\left(d^{\frac{1}{2}}\right)^3 = d^{\left(\frac{1}{2}\right)(3)} = d^{\frac{3}{2}}$

g. $\boldsymbol{x^{\frac{4}{3}}}$

$(x^4)^{\frac{1}{3}} = x^{4\left(\frac{1}{3}\right)} = x^{\frac{4}{3}}$

h. **3 ft**

$(27\text{ ft}^3)^{\frac{1}{3}} = 3\text{ ft}^{\frac{3}{3}} = 3\text{ ft}^1 = 3\text{ ft}$

i. **8**

$4^{\frac{3}{2}} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

Worksheet 2.7B

1.

a. $\boldsymbol{\sqrt{4^3}}$ *or* $\boldsymbol{\left(\sqrt{4}\right)^3}$ *or* $\boldsymbol{4^1\sqrt{4}}$

Note: While this could also be written as $4\sqrt{4}$, the directions said to use the square root symbol *and* a whole-number exponent.

b. **64**

$16^{\frac{3}{2}} = 4^3 = 64$

c. $\boldsymbol{x^{\frac{1}{2}}}$

$x^{\frac{4}{8}\div\frac{4}{4}} = x^{\frac{1}{2}}$

d. $\mathbf{9^{\frac{3}{2}}}$	
e. **27**	
$\left(\sqrt{9}\right)^3 = 27$	*Note:* Students should have found this both using a calculator (inputting $9^{\frac{3}{2}}$) and by hand (thinking through that $\sqrt{9} = 3$ and $3^3 = 27$). The point is to show them that inputting the fractional exponent into the calculator does work.
f. $\boldsymbol{b}$	
$b^{\frac{1}{2}}b^{\frac{1}{2}} = b^{\frac{2}{2}} = b^1 = b$	
g. $\boldsymbol{b}$	(A square root times itself equals the number under the square root sign.)
h. $\boldsymbol{b^0}$ ***or*** **1**	
$b^{\frac{1}{2}}b^{\frac{-1}{2}} = b^{\frac{0}{2}} = b^0$ *or* 1	
i. $\boldsymbol{b^{\frac{3}{2}}}$ ***or*** $\boldsymbol{b^1b^{\frac{1}{2}}}$	
$b^{\frac{1}{2}}b^{\frac{1}{2}}b^{\frac{1}{2}} = b^{\frac{3}{2}}$ *or* $b^1b^{\frac{1}{2}}$	
2.	
a. $\boldsymbol{y^{\frac{17}{3}}}$	
$y^{\frac{2}{3}}y^4y = y^{\frac{2}{3}+\frac{12}{3}+\frac{3}{3}} = y^{\frac{17}{3}}$	
b. $\boldsymbol{y^2}$	
$\left(y^{\frac{2}{3}}\right)^3 = y^{\left(\frac{2}{3}\right)(3)} = y^{\frac{2(3)}{3}} = y^{\frac{6}{3}} = y^2$	
c. $\boldsymbol{x^2}$	
$(x^6)^{\frac{1}{3}} = x^{6\left(\frac{1}{3}\right)} = x^{\frac{6}{3}} = x^2$	
d. $\boldsymbol{a^{\frac{-3}{2}}}$ ***or*** $\boldsymbol{a^{-\frac{3}{2}}}$	
$(a^{-2})^{\frac{3}{4}} = a^{-2\left(\frac{3}{4}\right)} = a^{\frac{-2(3)}{4}} = a^{\frac{-6}{4}} = a^{\frac{-3}{2}}$ *or* $a^{-\frac{3}{2}}$	
3.	
a. $\approx$ **12.699**	
$2^{\frac{11}{3}} \approx 12.699$	
b. $\approx$ **2.828**	
$2^{\frac{3}{2}} \approx 2.828$	
c. $\approx$ **2.828**	
$2^{\frac{3}{2}} \approx 2.828$	
d. $\approx$ **0.354**	
$2^{-\frac{3}{2}} \approx 0.354$	

4.

a. $\approx$ **41.569 mm²**	
$A = \frac{3\sqrt{3}}{2}(4\text{ mm})^2$	(substituted 4 mm for *s*)
$= \frac{3\sqrt{3}}{2}(16\text{ mm}^2)$	(squared 4 mm)
$\approx 41.569\text{ mm}^2$	(found 3 times the square root of 3 divided by 2; multiplied that by 16 mm²)
b. $\mathbf{\frac{3\left(3^{\frac{1}{2}}\right)}{2}s^2}$ ***or*** $\mathbf{\frac{3^{\frac{3}{2}}}{2}s^2}$	
$A = \frac{3\left(3^{\frac{1}{2}}\right)}{2}s^2$ *or* $A = \frac{3^{\frac{3}{2}}}{2}s^2$	
c. $\approx$ **2,599.208**	
$b_c = (800)2^{\frac{17\,hr}{10\,hr}} = (800)\left(2^{\frac{17\cancel{hr}}{10\cancel{hr}}}\right)$ $\approx 2{,}599.208$ bacteria	(used a calculator to complete the multiplication)
d. $\approx$ **0.0492 ft³**	
$85\text{ in}^3 = 85\,\cancel{\text{in}} \bullet \cancel{\text{in}} \bullet \cancel{\text{in}} \bullet \frac{1\text{ ft}}{12\,\cancel{\text{in}}} \bullet \frac{1\text{ ft}}{12\,\cancel{\text{in}}} \bullet \frac{1\text{ ft}}{12\,\cancel{\text{in}}} \approx 0.0492\text{ ft} \bullet \text{ft} \bullet \text{ft} \approx 0.0492\text{ ft}^3$	

Worksheet 2.8

1.

$\boldsymbol{x^{-7}}$	
$\frac{1}{x^7} = x^{-7}$	

2.

a. $\mathbf{7y^5}$	
$\frac{7y^3}{y^{-2}} = 7y^{3--2} = 7y^{3+2} = 7y^5$	
b. $\boldsymbol{b^{-3}}$	
$\left(b^{\frac{3}{4}}\right)^{-4} = b^{\frac{3(-4)}{4}} = b^{-3}$	
c. $\mathbf{3.0 \times 10^{-37}}$	
$(5.0 \times 10^{-8})(6.0 \times 10^{-30}) = 5.0(6.0) \times (10^{-8})(10^{-30}) = 30.0 \times 10^{-8+-30} = 30.0 \times 10^{-38}$ $= 3.0 \times 10^{-37}$	
d. $\boldsymbol{b^{\frac{7}{6}}}$	
$b^{\frac{1}{2}}b^{\frac{2}{3}} = b^{\frac{1}{2}+\frac{2}{3}} = b^{\frac{3}{6}+\frac{4}{6}} = b^{\frac{7}{6}}$	

e. $-\frac{3\text{ ft}}{\text{s}}$	
$\dfrac{\frac{18\text{ ft}^2}{\text{s}}}{-6\text{ ft}} = \frac{18\text{ ft}^2}{\text{s}} \bullet \frac{1}{-6\text{ ft}} = \frac{18\text{ ft}^2}{-6\text{ ft} \bullet \text{s}} = -\frac{3\text{ ft}^2}{\text{ft} \bullet \text{s}} = -\frac{3\text{ ft}^{2-1}}{\text{s}} = -\frac{3\text{ ft}}{\text{s}}$	
f. x	
$\sqrt{x}\sqrt{x} = x$	
g. $x^{\frac{-3}{4}}$	
$\frac{1}{x^{\frac{3}{4}}} = x^{\frac{-3}{4}}$	
h. $-a^2$	
$-(-a)^2 = -(-a)(-a) = -a^2$	
i. $\approx \pm 4.796$	
$\pm\sqrt{23} \approx \pm 4.796$	
j. ≈ 4.796	
$\sqrt{23} \approx 4.796$	(The $\sqrt{\ }$ notation is defined as meaning the positive root.)
k. ≈ 7.071	
l. ≈ 2.508	

3.

a. $280\frac{\text{kg} \bullet \text{m}^2}{\text{s}^2}$ ***or* 280 J**	
$W = \left(7\text{ kg} \bullet \frac{\text{m}}{\text{s}^2}\right)(40\text{ m}) = 280\frac{\text{kg} \bullet \text{m}^2}{\text{s}^2}$ *or* 280 J	
b. 7432.243 cm^2	
$8\text{ ft}^2 = 8\text{ ft} \bullet \text{ft} \bullet \frac{30.48\text{ cm}}{1\text{ ft}} \bullet \frac{30.48\text{ cm}}{1\text{ ft}} \approx 7432.243\text{ cm} \bullet \text{cm} = 7432.243\text{ cm}^2$	
c. $2{,}880\text{ in}^2$	
Converting 4 ft to in:	
$4\text{ ft}\left(\frac{12\text{ in}}{1\text{ ft}}\right) = 48\text{ in}$	
Finding the area:	
$A = lw$	(formula for the area of a rectangle; we could find this in Appendix B)
$A = (48\text{ in})(60\text{ in}) = 2{,}880\text{ in}^2$	

Chapter 3 Numbers and Sets

Worksheet 3.1A

1.

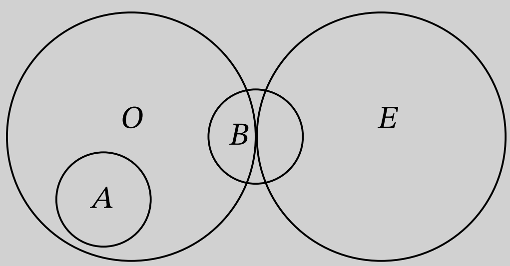
a. **Answers will vary. Students just need to list 3 pile types.**	Possibilities include pencils, pens, writing utensils, sticky notes, paper clips, keys, etc.
b. $\in$	

$p \in P$

Worksheet 3.1B

1. *Note:* The capitalization of set names matters.

a.–c.

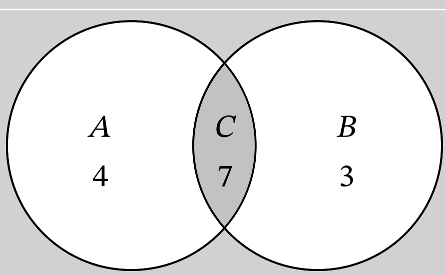


(The two large circles are drawn separately, with no overlap, as a number is either odd or even, but never both. Students were told to use O to stand for odd and E for even. Students should have drawn a small circle labeled A inside the O circle, since {5, 11, 15} are all odd numbers. Students should have drawn a small circle labeled B with part inside O and part inside E since {5, 6, 10, 15} are both in set O and set E.)

2.

a. $\in$	
$6 \in B$	
b. $\notin$	
$y \notin \mathrm{B}$	
c. **{3, 4, 7}**	(We combined both set A and set B, including every element in either set.)
$C = \{3, 4, 7\}$	
d. **{7}**	(We looked only for the elements that were in both the sets: 7.)
$C = \{7\}$	
e. **Yes. $D \subset A$**	(D is a subset of set A, as all of its elements are also part of set A.)

f.



(The numbers do not have to be on the diagram, but are given to show why the diagram is drawn the way it is.)

g.	**∅**	(A and B don't share any elements, so there is nothing in their intersection.)
	$A \cap B = \varnothing$	
h.	$\boldsymbol{F \subset S}$ *or* $\boldsymbol{F \subseteq S}$	(We could also write $S \supset F$ to show that S is a set that contains F, but this notation was not taught.)

3.

a.	$\boldsymbol{x^{\frac{3}{2}}}$	
	$\left(x^{\frac{1}{4}}\right)^6 = x^{\left(\frac{1}{4}\right)(6)} = x^{\frac{6}{4}} = x^{\frac{3}{2}}$	
b.	$\boldsymbol{x^{\frac{4}{3}}}$	
	$x^{\frac{1}{3}}x = x^{\frac{1}{3}}x^{\frac{3}{3}} = x^{\frac{4}{3}}$	
c.	$\boldsymbol{x^7}$	
	$\frac{x^4x^{-2}}{x^{-5}} = x^{4+-2--5} = x^{4+-2+5} = x^7$	
d.	$\boldsymbol{\frac{2cx+3b}{bc}}$	
	$\frac{2x}{b} + \frac{3}{c} = \frac{2x}{b}\left(\frac{c}{c}\right) + \frac{3}{c}\left(\frac{b}{b}\right) = \frac{2cx}{bc} + \frac{3b}{bc} = \frac{2cx+3b}{bc}$	
e.	$\mathbf{7{,}204\frac{m}{min^2}}$	
	$2\frac{\text{m}}{\text{s}^2} + 4\frac{\text{m}}{\text{min}^2} = \frac{2\text{ m}}{\cancel{\text{s}^2}}\left(\frac{60\ \cancel{\text{s}}}{1\text{ min}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\text{ min}}\right) + 4\frac{\text{m}}{\text{min}^2} = 7{,}200\frac{\text{m}}{\text{min}^2} + 4\frac{\text{m}}{\text{min}^2} = 7{,}204\frac{\text{m}}{\text{min}^2}$	
f.	**4**	
	$\sqrt{4}\sqrt{4} = 4$	
g.	$\boldsymbol{x^4}$	
	$(-x^2)^2 = x^{2(2)} = x^4$	

4.	$\mathbf{1.381 \times 10^{-23}\ J \bullet K^{-1}}$	

5.

a.	$\boldsymbol{V = k_B NTP^{-1} = \frac{k_B NT}{P^1}}$	(rewrote the P^{-1} as P^1 in the denominator)
b.	$\mathbf{\frac{\left(1.512 \times 10^{26}\right)\left(200\ K\right)\left(1.381 \times 10^{-23}\frac{J}{K}\right)}{100\frac{J}{m^3}}}$	(substituted values given)

c. $\dfrac{\mathbf{4.176 \times 10^3\ K \bullet \frac{J}{K}}}{\mathbf{\frac{J}{m^3}}}$	
$\dfrac{(1.512)(1.381)(200) \times (10^{26})(10^{-23})\ \text{K} \bullet \frac{\text{J}}{\text{K}}}{100\frac{\text{J}}{\text{m}^3}}$	(grouped known values and units together)
$\dfrac{417.614 \times 10^{26+-23}\ \text{K} \bullet \frac{\text{J}}{\text{K}}}{100\frac{\text{J}}{\text{m}^3}}$	(multiplied known values)
$\dfrac{4.176 \times 10^3\ \text{K} \bullet \frac{\text{J}}{\text{K}}}{\frac{\text{J}}{\text{m}^3}}$	(simplified exponent of 10 and divided by the 100 in the denominator and rounded to the third decimal)
d. $\mathbf{4.176 \times 10^3\ m^3}$	
$4.176 \times 10^3 \bullet \text{K} \bullet \frac{\text{J}}{\text{K}} \bullet \frac{\text{m}^3}{\text{J}}$	(inverted the denominator and multiplied to complete the division)
$4.176 \times 10^3 \bullet \cancel{\text{K}} \bullet \frac{\cancel{\text{J}}}{\cancel{\text{K}}} \bullet \frac{\text{m}^3}{\cancel{\text{J}}} = 4.176 \times 10^3\ \text{m}^3$	(simplified the units)

Worksheet 3.2A

1.

a. **No**	($x \in \mathbb{N}$ tells us that x must be a natural number, and 1.56 is not a natural number, even though it is less than 8. The natural numbers less than 8 are 1, 2, 3, 4, 5, 6, and 7)
b. $\boldsymbol{x} \in \mathbb{C}$	
c. **No**	(4 is not in the set specified.)
d. **No**	(5 is a natural number, and $x \notin \mathbb{N}$ tells us that x cannot be a natural number.)
e. **True**	Rational numbers ($\mathbb{Q}$) are a subset of real numbers ($\mathbb{R}$).

Worksheet 3.2B

1.

a. **Real and Rational** *or* $\mathbb{R}$ and $\mathbb{Q}$

b. $y \in \mathbb{N} \cap y > 3$

c. **$z \in \mathbb{Q}$ means that z must be a rational number *or* z is an element of the rational numbers.**

d. **It means that the union of the set of rational numbers $(\mathbb{Q})$ and the set of irrational numbers $(\mathbb{P})$ equals the set of real numbers $(\mathbb{R})$** — in other words, that if you take rational numbers and irrational numbers, you get the set of all real numbers.

e. **It means that the intersection of the set of rational numbers $(\mathbb{Q})$ and the set of irrational numbers $(\mathbb{P})$ is empty**; there aren't any numbers that are both rational and irrational.

2. *Note:* These values are listed in the text for Lesson 3.2.

a. $\boldsymbol{\pi \approx 3.14159}$

b. $\boldsymbol{e \approx 2.71828}$

c. $\boldsymbol{\phi \approx 1.61803}$

d. **ρ is pronounced "rho."**

3.

a. $\mathbf{8.992 \times 10^9 \frac{Nm^2}{C^2}}$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$k = \frac{1}{4(3.14159)\left(8.85 \times 10^{-12}\frac{\mathrm{C}^2}{\mathrm{N \bullet m}^2}\right)} \approx \frac{1}{111.212286 \times 10^{-12}\frac{\mathrm{C}^2}{\mathrm{N \bullet m}^2}} \approx$$

$$\frac{1}{1.112 \times 10^{-10}\frac{\mathrm{C}^2}{\mathrm{N \bullet m}^2}} \approx 8.992 \times 10^9 \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$$ (simplified so rounded and in scientific notation)

Note: The answer could also be written as $\frac{1}{1.112 \times 10^{-10}}\frac{\mathrm{N \bullet m}^2}{\mathrm{C}^2}$, as we could invert and multiply the $\frac{\mathrm{C}^2}{\mathrm{N \bullet m}^2}$ part, giving the answer as $8.992 \times 10^9 \frac{\mathrm{N \bullet m}^2}{\mathrm{C}^2}$.

b. **53.919 Volts *or* 53.919 V**

$$V_c = V_s e^{\frac{-t}{RC}}$$

$$V_c = (120 \text{ Volts})\left(e^{\frac{-4\,\mathrm{s}}{5\,\mathrm{s}}}\right) = (120 \text{ Volts})\left(e^{\frac{-4}{5}}\right) = 53.919 \text{ Volts } \textit{or } 53.919 \text{ V}$$

Note: Students may also have gotten 53.920 if they used a more rounded value of *e*.

4.

a. $\boldsymbol{b^{-\frac{4}{3}}}$

$$\left(b^{\frac{2}{3}}\right)^{-2} = b^{\left(\frac{2}{3}\right)(-2)} = b^{-\frac{4}{3}}$$

b. $\boldsymbol{b^4}$

$(-b)^4 = (-b)(-b)(-b)(-b) = b^4$

c. $\boldsymbol{-\frac{8a + 3x}{ax}}$

$-\left(\frac{8}{x} + \frac{3}{a}\right) = -\left(\frac{8}{x}\left(\frac{a}{a}\right) + \frac{3}{a}\left(\frac{x}{x}\right)\right) = -\left(\frac{8a}{ax} + \frac{3x}{ax}\right) = -\frac{8a + 3x}{ax}$

d. $\boldsymbol{-x^{-7}}$

$-\frac{x^2}{x^9} = -x^{2-9} = -x^{-7}$

e. $\boldsymbol{a^6}$

$- - a^8a^{-2} = a^8a^{-2} = a^{8+-2} = a^6$

f. **16**

$\sqrt{16}\sqrt{16} = 16$

g. $\boldsymbol{y^{\frac{7}{3}}}$

$y^{-\frac{2}{3}}y^3 = y^{-\frac{2}{3}+\frac{9}{3}} = y^{\frac{7}{3}}$

Worksheet 3.3A

1.

a. $\boldsymbol{2\sqrt{3}}$

$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

b. $\boldsymbol{4\sqrt{2}}$

$32^{\frac{1}{2}} = \sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

c. $\boldsymbol{3\sqrt{5}}$

$45^{\frac{1}{2}} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

2.

a. $\boldsymbol{\sqrt{6}}$

$\sqrt{2}\sqrt{3} = \sqrt{6}$

b. $\boldsymbol{\sqrt{-5}}$

$\sqrt{-1}\sqrt{5} = \sqrt{-5}$

c. $\sqrt{10}$

$\frac{\sqrt{100}}{\sqrt{10}} = \sqrt{\frac{100}{10}} = \sqrt{10}$

d. $\sqrt[4]{4}$

$\frac{8^{\frac{1}{4}}}{2^{\frac{1}{4}}} = \left(\frac{8}{2}\right)^{\frac{1}{4}} = 4^{\frac{1}{4}} = \sqrt[4]{4}$

Worksheet 3.3B

1.

a. $\mathbf{3\sqrt{6}}$

$54^{\frac{1}{2}} = \sqrt{54} = \sqrt{9}\sqrt{6} = 3\sqrt{6}$

b. $\mathbf{4\sqrt{x}}$

$(16x)^{\frac{1}{2}} = \sqrt{16x} = 4\sqrt{x}$

2.

a. **=**	$\left(\frac{\sqrt{16}}{\sqrt{2}} = \sqrt{\frac{16}{2}} = \sqrt{8}\right)$
b.	(In the fraction on the right, the numerator is raised to the $\frac{1}{2}$ and the denominator to the $\frac{1}{3}$; since the roots are different, we can't divide them the way we could if they were the same; we would have to calculate each root. In doing so, we would see it doesn't equal the expression on the left.)
c. **=**	$\left(\sqrt{x^3} = \sqrt{x^2}\sqrt{x} = x\sqrt{x}\right)$

3.

a. $\mathbf{\sqrt{\frac{1}{2}}}$

$\frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}}$

b. $\mathbf{\sqrt{5}}$

$\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{5}$

4.

a. **3**

b. ***a***

c. ***a***

5. $\mathbf{\sqrt{A_l A_w}}$

$x = \sqrt{A_l}\sqrt{A_w} = \sqrt{A_l A_w}$

6.

a. **{25}**

$A \cap B = \{25\}$	(25 is the only element both sets share.)
b. **No**	(*s* must be a natural number, and $\frac{5}{6}$ is not a natural number.)

c. $\boldsymbol{x^{\frac{13}{2}}}$

$(-x)^2 x^3 x x^{\frac{1}{2}} = (-x)(-x)x^3 x x^{\frac{1}{2}} = xx x^3 x x^{\frac{1}{2}} = x^6 x^{\frac{1}{2}} = x^{\frac{13}{2}}$

d. $\boldsymbol{x^5}$

$-\frac{-(x^3)^2}{x} = \frac{(x^3)^2}{x} = \frac{x^6}{x} = x^{6-1} = x^5$

Worksheet 3.4A

1.

a. $\boldsymbol{\sqrt{16}\sqrt{-1}}$

b. $\boldsymbol{\sqrt{9}\sqrt{-1}}$

c. $\boldsymbol{\sqrt{x^2}\sqrt{-1}}$

2.

a. $\boldsymbol{4i}$

b. $\boldsymbol{3i}$

c. $\boldsymbol{xi}$

Worksheet 3.4B

1.

a. $\boldsymbol{\sqrt{17}i}$

$\sqrt{-17} = \sqrt{17}\sqrt{-1} = \sqrt{17}i$

b. $\boldsymbol{ai}$

$\sqrt{-a^2} = \sqrt{a^2}\sqrt{-1} = ai$

c. $\boldsymbol{3\sqrt{2}i}$

$\sqrt{-18} = \sqrt{3}\sqrt{3}\sqrt{2}\sqrt{-1} = 3\sqrt{2}i$

d. $\boldsymbol{\sqrt{x}i}$

$\sqrt{-x} = \sqrt{x}\sqrt{-1} = \sqrt{x}i$

e. $\boldsymbol{2\sqrt{6}i}$

$(-24)^{\frac{1}{2}} = \sqrt{-24} = \sqrt{4}\sqrt{6}\sqrt{-1} = 2\sqrt{6}i$

2. *Note:* Make sure the real numbers are written first and then the imaginary.

a. $\boldsymbol{0 + \sqrt{17}i}$

b. $\boldsymbol{0 + ai}$

c. $\boldsymbol{0 + 3\sqrt{2}i}$

d. $\boldsymbol{0 + \sqrt{x}i}$

e. $\boldsymbol{0 + 2\sqrt{6}i}$

3. *Note:* Make sure the real numbers are written first and then the imaginary.

a. **$4 + 2i$**

b. **$7 + 4i$**

c. **$2 - 2i$**

d. **$-4 - 2i$**

e. **$-5 + 2i$**

4. *Note:* We rewrote as a fractional exponent so we could use the calculator.

a. **-3**

$\sqrt[3]{-27} = (-27)^{\frac{1}{3}} = -3$

b. **≈ -3.803**

$\sqrt[3]{-55} = (-55)^{\frac{1}{3}} \approx -3.803$

c. **≈ -3.634**

$\sqrt[3]{-48} = (-48)^{\frac{1}{3}} \approx -3.634$

5.

a. **Yes**	(x can be either an integer *or* an imaginary number, and $4i$ is an imaginary number.)
b. **Yes**	($\mathbb{Q}$ [rational numbers] is a subset of $\mathbb{R}$ [real numbers].)
c. **$\{d\}$**	
$A \cap B = \{d\}$	(d is the only element in both sets.)
d. **$\{a, b, c, d, e\}$**	
$A \cup B = \{a, b, c, d, e\}$	(listed all elements in either set)

Worksheet 3.5A

1.

a. **-24**

$8i(3i) = 24i^2 = 24\sqrt{-1}\sqrt{-1} = 24(-1) = -24$

b. **24**

$24i^4 = 24\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1} = 24(-1)(-1) = 24$

c. **$24i$**

$24i^5 = 24\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1} = 24(-1)(-1)\sqrt{-1} = 24i$

d. **-24**

$24i^6 = 24\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1} = 24(-1)(-1)(-1) = -24$

Worksheet 3.5B

1.

a. $\boldsymbol{5i^{-1}}$ ***or*** $\boldsymbol{\frac{5}{i}}$ ***or*** $\boldsymbol{-5i}$

$\frac{5i^2}{i^3} = 5i^{2-3} = 5i^{-1}$ *or* $\frac{5}{i}$

Note: Some students might notice they can also multiply this by $\frac{i}{i}$ and simplify to $-5i$ which is also correct since you can always multiply by a fraction worth $1 = \frac{i}{i}$; see 2e below.

b. $\boldsymbol{3i}$

$3i^3i^{-2} = 3i^{3+-2} = 3i$

c. $\boldsymbol{-3}$

$\frac{6i^5}{2i^3} = 3i^{5-3} = 3i^2 = 3\sqrt{-1}\sqrt{-1} = 3(-1) = -3$

d. **4**

$-4i^2 = -4\sqrt{-1}\sqrt{-1} = -4(-1) = 4$

2.

a. $\boldsymbol{-8\sqrt{10}}$

$\sqrt{-40}\sqrt{-16} = \sqrt{40}i\sqrt{16}i = \sqrt{4}\sqrt{10}\sqrt{16}i^2 = 2\sqrt{10}(4)(-1) = -8\sqrt{10}$

b. $\boldsymbol{4i}$

$\sqrt{-8}\sqrt{2} = \sqrt{8}i\sqrt{2} = \sqrt{4}\sqrt{2}\sqrt{2}i = 2(2)i = 4i$

c. $\boldsymbol{-9}$

$\sqrt{-3}\sqrt{-27} = \sqrt{3}i\sqrt{27}i = \sqrt{3}\sqrt{9}\sqrt{3}i^2 = 3(3)(-1) = -9$

d. **2**

$\frac{\sqrt{-8}}{\sqrt{-2}} = \frac{\sqrt{8}i}{\sqrt{2}i} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$

(The *i*'s cancel out.)

e. $\boldsymbol{-\sqrt{2}i}$

$\frac{\sqrt{6}}{\sqrt{-3}} = \frac{\sqrt{6}}{\sqrt{3}i} = \sqrt{\frac{6}{3}}\left(\frac{1}{i}\right) = \sqrt{2}\left(\frac{1}{i}\right) = \frac{\sqrt{2}}{i} = \frac{\sqrt{2}}{i}\left(\frac{i}{i}\right) = \frac{\sqrt{2}i}{i^2} = \frac{\sqrt{2}i}{-1} = -\sqrt{2}i$

Note: Students may also have listed their answer as $\frac{\sqrt{2}}{i}$; if so, this is correct, but show them how they could simplify by multiplying by $\frac{i}{i}$; the convention is to avoid leaving *i* in the denominator.

f. $\boldsymbol{\sqrt{\frac{1}{2}}i}$ ***or*** $\boldsymbol{\sqrt{0.5}i}$

$\frac{\sqrt{-7}}{\sqrt{14}} = \frac{\sqrt{7}i}{\sqrt{14}}\sqrt{\frac{7}{14}}i = \sqrt{\frac{1}{2}}i$ *or* $\sqrt{0.5}i$

3.	**{ 8.5 in, 11 in, 17 in}**	
	$A \cup B = \{ 8.5 \text{ in}, 11 \text{ in}, 17 \text{ in}\}$	(We included all the elements in both A or B in the union.)
4.		
a.	$\boldsymbol{\frac{x^3 y + 10x}{5}}$	
	$\frac{x^3y}{5} + 2x = \frac{x^3y}{5} + 2x\left(\frac{5}{5}\right) = \frac{x^3y}{5} + \frac{10x}{5} = \frac{x^3y + 10x}{5}$	
b.	$\boldsymbol{\frac{6x^4 y - ab}{3b}}$	
	$\frac{2x^4y}{b} + \frac{-a}{3} = \frac{2x^4y}{b}\left(\frac{3}{3}\right) + \frac{-a}{3}\left(\frac{b}{b}\right) = \frac{6x^4y}{3b} + \frac{-ab}{3b} = \frac{6x^4y - ab}{3b}$	
c.	$\boldsymbol{x^{-3} \text{ or } \frac{1}{x^3}}$	
	$-\frac{x^3x^{-2}}{-x^4} = \frac{x^3x^{-2}}{x^4} = x^3x^{-2}x^{-4} = x^{-3} \text{ or } \frac{1}{x^3}$	(the opposite of the opposite is a positive, so the negative signs canceled out)
d.	$\boldsymbol{-y^5}$	
	$(-y)^5 = (-y)(-y)(-y)(-y)(-y) = -y^5$	
e.	$\boldsymbol{y^6}$	
	$(-y)^6 = (-y)(-y)(-y)(-y)(-y)(-y) = y^6$	
f.	**25**	
g.	***b***	(A square root times itself, whether that square root is positive or negative, will equal the number under the square root sign . . . even if that number is an unknown.)
h.	$\boldsymbol{y^{\frac{31}{6}}}$	
	$y^{\frac{2}{3}}y^4y^{\frac{1}{2}} = y^{\frac{4}{6}}y^{\frac{24}{6}}y^{\frac{3}{6}} = y^{\frac{31}{6}}$	
i.	$\boldsymbol{y^{\frac{8}{3}}}$	
	$\left(y^{\frac{2}{3}}\right)^4 = y^{\frac{2(4)}{3}} = y^{\frac{8}{3}}$	

Worksheet 3.6

1.

a. $\mathbb{P}$

$\mathbb{R} - \mathbb{Q} \equiv \mathbb{P}$

b. **{1, 3, 5, 7}**

$A \cup B = \{1, 3, 5, 7\}$

c. **{3, 5}**

$A \cap B = \{3, 5\}$

d. $\boldsymbol{y \in \mathbb{R} \cap y < 2}$

e. $\boldsymbol{3\sqrt{7}i}$

$\sqrt{-63} = \sqrt{63}i = \sqrt{9}\sqrt{7}i = 3\sqrt{7}i$

f. $\boldsymbol{\sqrt{10}i}$

$$\frac{\sqrt{-70}}{\sqrt{7}} = \frac{\sqrt{70}i}{\sqrt{7}} = \sqrt{\frac{70}{7}}i = \sqrt{10}i$$

g. $\boldsymbol{\left(\frac{x}{b}\right)^{\frac{1}{4}}}$

$$\frac{x^{\frac{1}{4}}}{b^{\frac{1}{4}}} = \left(\frac{x}{b}\right)^{\frac{1}{4}}$$

h. $\boldsymbol{yi}$

$\sqrt{y}\sqrt{-y} = \sqrt{y}\sqrt{y}i = yi$

i. $\boldsymbol{5i}$

$(-25)^{\frac{1}{2}} = \sqrt{-25} = 5i$

j. $\boldsymbol{-\frac{3}{2}}$

$$3i(i^2)\left(\frac{1}{2i}\right) = \frac{3i^3}{2i} = \frac{3}{2}i^2 = -\frac{3}{2}$$

k. $\boldsymbol{-6}$

$\sqrt{-18}\sqrt{-2} = \sqrt{18}i\sqrt{2}i = \sqrt{9}\sqrt{2}\sqrt{2}i^2 = 3(2)(-1) = -6$

2. *Note:* These problems were designed to give students an opportunity to engage with different notations, which is something they'll have to do regularly if they pursue a career in a scientific or mathematical field. However, the point is to simply make them think about different ways of expressions; it is okay if they struggled with these problems (they do not have to know these notations).

a. ***x* is an imaginary number *or x* such that *x* is an element of imaginary numbers *or x* is an element of imaginary numbers *or*** $\boldsymbol{x \in \mathbb{I}}$.

b. $\boldsymbol{-2 \le x < 3}$ ***or* a value greater than or equal to –2 but less than 3**

c. $\boldsymbol{5 < x \le 10}$ ***or* a value greater than or equal to 5 but less than or equal to 10**

Chapter 4 Solving for the Unknown

Worksheet 4.1

1.

a. $\boldsymbol{x = 3}$	
$9 = 3x$	(original equation)
$3 = x$	(divided both sides by 3)
b. $\boldsymbol{x = -3}$	
$9 = -3x$	(original equation)
$-3 = x$	(divided both sides by –3)
c. $\boldsymbol{x = 10}$	
$-120 = -12x$	(original equation)
$10 = x$	(divided both sides by –12)
d. $\boldsymbol{x = 9}$	
$15 = x + 6$	(original equation)
$9 = x$	(subtracted 6 from both sides)
e. $\boldsymbol{x = \frac{4}{5}}$	
$\frac{2}{5} = \frac{1}{2}x$	(original equation)
$\frac{2}{5}\left(\frac{2}{1}\right) = x$	(multiplied both sides by $\frac{2}{1}$, as the inverse of $\frac{1}{2}$ can also be thought of as dividing both sides by $\frac{1}{2}$, only jumping to the inverting and multiplying step)
$\frac{4}{5} = x$	(simplified)
f. $\boldsymbol{x = -\frac{17}{12}}$	
$\frac{3}{4}i^2 = x + \frac{2}{3}$	(original equation)
$-\frac{3}{4} = x + \frac{2}{3}$	(simplified i^2 to –1)
$-\frac{3}{4} - \frac{2}{3} = x$	(subtracted $\frac{2}{3}$ from both sides)
$-\frac{9}{12} - \frac{8}{12} = x$	(rewrote with common denominator)
$-\frac{17}{12} = x$	(simplified)

g. $\boldsymbol{x = 4}$	
$\frac{1}{2}x^6x^{-5}x^{\frac{1}{3}}x^{-\frac{1}{3}} = 2$	(original equation)
$\frac{1}{2}x^{6-5+\frac{1}{3}-\frac{1}{3}} = 2$	(simplified exponents)
$\frac{1}{2}x^1 = 2$	(simplified exponents)
$x = 4$	(multiplied both sides by 2)
h. $\boldsymbol{x = -4}$	
$\sqrt{-16}\sqrt{-25} = 5x$	(original equation)
$\sqrt{16}i\sqrt{25}i = 5x$	(thought of $\sqrt{-16}$ as $\sqrt{16}\sqrt{-1}$ and $\sqrt{-25}$ as $\sqrt{25}\sqrt{-1}$ and wrote $\sqrt{-1}$ as i)
$4i5i = 5x$	(found square roots)
$20i^2 = 5x$	(simplified multiplication)
$-20 = 5x$	(simplified the left side of the equation)
$-4 = x$	(divided both sides by 5)

Note: Students should have checked their work in problem 1 by substituting their answers back into the original equations. If they got any problems incorrect, talk to them about whether they really checked their work.

2. *Note:* Watch to make sure students included the correct unit of measure in their answer.

a. $\boldsymbol{F = 5\frac{\textbf{kg} \bullet \textbf{m}}{\textbf{s}^2}}$	
$P = Fv$	(relationship given)
$\frac{P}{v} = F$	(divided both sides by v)
$\frac{50 \text{ kg}\frac{\text{m}^2}{\text{s}^2}}{10\frac{\text{m}}{\text{s}}} = F$	(inserted known values)
$5\frac{\frac{\text{kg} \bullet \text{m}^2}{\text{s}^2}}{\frac{\text{m}}{\text{s}}} = F$	(divided known values and rewrote the units in the numerator as a single fraction)
$5\frac{\text{kg} \bullet \text{m}^2}{\text{s}^2}\left(\frac{\text{s}}{\text{m}}\right) = F$	(inverted and multiplied)
$5\frac{\text{kg} \bullet \text{m}}{\text{s}} = F$	(simplified units)